DSGE models for developing economies: an application to Morocco

Mohammed Ait Lahcen

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DSGE models for developing economies: an application to Morocco

Author:
Mohammed Ait Lahcen

Supervisor:
Prof. Kenza Benhima

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Declaration of Authorship

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- This work was done wholly or mainly while in candidature for a Master degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
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Abstract

Faculty of Business and Economics - HEC
Department of Economics - DEEP

Master of Science in Economics

DSGE models for developing economies: an application to Morocco

by Mohammed Ait Lahcen

In this thesis we try to understand the impact of some macroeconomic features of developing economies, in particular the existence of a large informal sector, on the reaction of these economies to different shocks. In order to achieve this objective, we derive a simple New Keynesian Small Open Economy DSGE model featuring multiple sectors with monopolistic competition, nominal rigidities in prices, a fixed exchange regime and the introduction of a simple medium-sized informal sector. We estimate the model with Bayesian estimation using quarterly data from Morocco.

The model does a good job in capturing the unconditional second moments of the data. It is also able to replicate well some of the historical data series. Estimation results suggest a relatively weaker role of price rigidities in the non-tradables sector. It also suggests a much more aggressive reaction of the central bank to inflationary pressures with a relatively higher weight given to fluctuations in inflation compared with fluctuations in output and the real exchange rate. The study of the Bayesian impulse-response functions confirm the shock absorbing role of the informal sector for productivity shocks and pleads towards excluding imported inflation from the inflation target. However, no evidence is found of a shock absorbing role of the informal sector in the case of interest rate or foreign demand shocks.

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To my parents

Chapter 1

Introduction

From a macroeconomic point of view, developing economies tend to differ from developed economies in some important features such as the existence of a large informal sector, vulnerability to external shocks and a weak financial sector. Such particularities generate different reactions in response to the same shocks. Trying to apply the exact same models created for developed countries to the study of developing countries is a pointless exercise which may result in wrong conclusions and lead to counter-productive policy recommendations. From this comes the need to build structural macroeconomic models for developing economies which take into account at least some of these particularities when drawing from the existing literature.

According to Ahmed et al. (2012), some of the common features of developing countries that should be taken into consideration are:

- Small open economy;
- Vulnerability to external shocks;
- Existence of a significant underground economy;
- Weak financial sector;
- Weak economic and political institutions;

The objective of this thesis is to study the impact of some of these specific features on the reaction of the economy to different shocks. In particular, we want to know how the presence of a relatively important informal sector affects the behavior of some macroeconomic variables in response to exogenous shocks in a developing economy characterized by capital controls and a fixed exchange rate regime.

To answer this question, we develop a simple New Keynesian small open economy DSGE model featuring fixed exchange regime, nominal rigidities, external shocks and a medium-sized informal sector both in the labor and product markets. We will try to answer our question by studying the interactions of the main macroeconomic variables with different shocks through the analysis of the impulse-response functions produced by the model.

To make the model more representative of developing countries, we calibrate and estimate its parameters based on quarterly data for the Moroccan economy. Choosing Morocco as a subject of our research is motivated by the fact that it features all the typical characteristics of developing economies cited above. Quarterly data availability was also a decisive factor. DSGE literature based on data from Morocco is virtually non-existent which increases the interest of this work.

The thesis is organized as follows: the next chapter is a brief review of DSGE literature for developing countries with a particular focus on informality. Chapter 3 presents some of the stylized facts of the Moroccan economy as well as the place of the informal sector. The purpose of this part is to justify the modeling choices introduced later. Chapter 4 presents the model and its main theoretical features. Chapter 5 describes the calibration and estimation of the model's parameters and compares the model's results with data. The model will be calibrated using long-term averages from data as well as standard parameter values used in the literature. Estimation of the remaining parameters will be done using Bayesian estimation approach based on quarterly data of the Moroccan economy with a thoroughly documented choice of prior distributions. The resulting posterior distributions will be compared to the priors. The empirical fit of the model will be tested through the study of the Bayesian impulse-response functions resulting from 5 exogenous shocks, historical simulations as well as the comparison of the model generated theoretical second moments and the empirical unconditional second moments taken from the data.

Chapter 2

Literature review

In the following, we present a brief survey of the DSGE literature on developing economies. We focus mainly on authors trying to incorporate the informal sector into a DSGE framework and present the results most relevant to our work.

Assuming capital mobility between the formal and informal sectors in a two-sector small open economy GE model, Marjit and Kar (2009) show that trade liberalization in the formal sector may benefit informal workers by raising both wage and employment in the informal sector.

Using a simple dynamic model, Ihrig and Moe (2004) try to study the evolution of the informal sector. They find that the size of the informal sector is negatively affected by a reduction in the tax rate.

Conesa et al. (2002) introduce an informal goods sector into a Real Business Cycle model in order to explain cross-country differences in aggregate fluctuations. Production technology in the informal sector is different but the goods produced are tradable across sectors. The formal sector pays an exogenously specified wage premium over the informal sector but informal workers enjoy more leisure time. According to their model, the size of the underground economy is negatively related to the size of the wage premium between the two sectors. The existence and size of the informal sector affect the decision of workers to change sectors following a productivity shock. This mobility across sectors amplifies the response of formal output to external shocks which leads to a negative correlation between participation rate and aggregate fluctuations. The smaller the participation rate, the bigger the informal sector and the higher the effect of technological shocks.

Fiess et al. (2007) examine the adjustment of labor markets in developing countries to exogenous shocks by introducing a two-sector labor market into a small open economy

model. The labor market is divided into a formal tradable sector and an informal non-tradable sector. The authors study the co-movements between the relative sizes and earnings of the two sectors. They show that co-movements between the two sectors are a function of the existence of wage rigidities in the formal sector and also on the origin of the external shocks.

Castillo and Montoro (2008) build a New-Keynesian model to study the effect of informal labor markets on the dynamics of inflation and the transmission of monetary policy. the authors use a search and matching framework to generate frictions in the labor market. They show that the underground economy works as a buffer that weakens the effect of external demand shocks on aggregate wages and inflation. According to their model, during high demand periods firms tend to increase output by taping into the cheaper informal labor markets thus avoiding pressure on formal wages.

Ahmed et al. (2012) develop a closed economy New-Keynesian model with informality in the labor and production markets. In contrast with formal sector firms, firms in the informal goods sector use only labor in their production technology and operate in a perfectly competitive market. The authors introduce frictions in the formal labor market by modeling formal labor as a composite of labor differentiated on basis of skill level. In a subsequent paper, Ahmed et al. (2013) estimate the same model using Bayesian estimation and quarterly data of Pakistan economy. They find evidence of the absorbing role of the informal sector in the short run in response to interest rate shocks.

Haider et al. (2012) develop a small open economy New-Keynesian model. They include several features specific to developing countries such as foreign direct investments, oil imports and informal labor and production sectors. In the informal production sector, firms do not pay taxes and are less productive compared to formal sector firms. Households provide labor to both formal and informal sectors and enjoy some market power to set wages based on their skills in the formal labor market. The authors assert that including those features increases the economy's exposure to internal and external shocks in line with the stylized facts of developing countries business cycles. Haider et al. (2012) calibrate the model for Pakistan economy and evaluate alternative monetary policies. Their results suggest monetary authorities in developing countries should stick to the Taylor principle and put some weight on exchange rate fluctuations.

Chapter 3

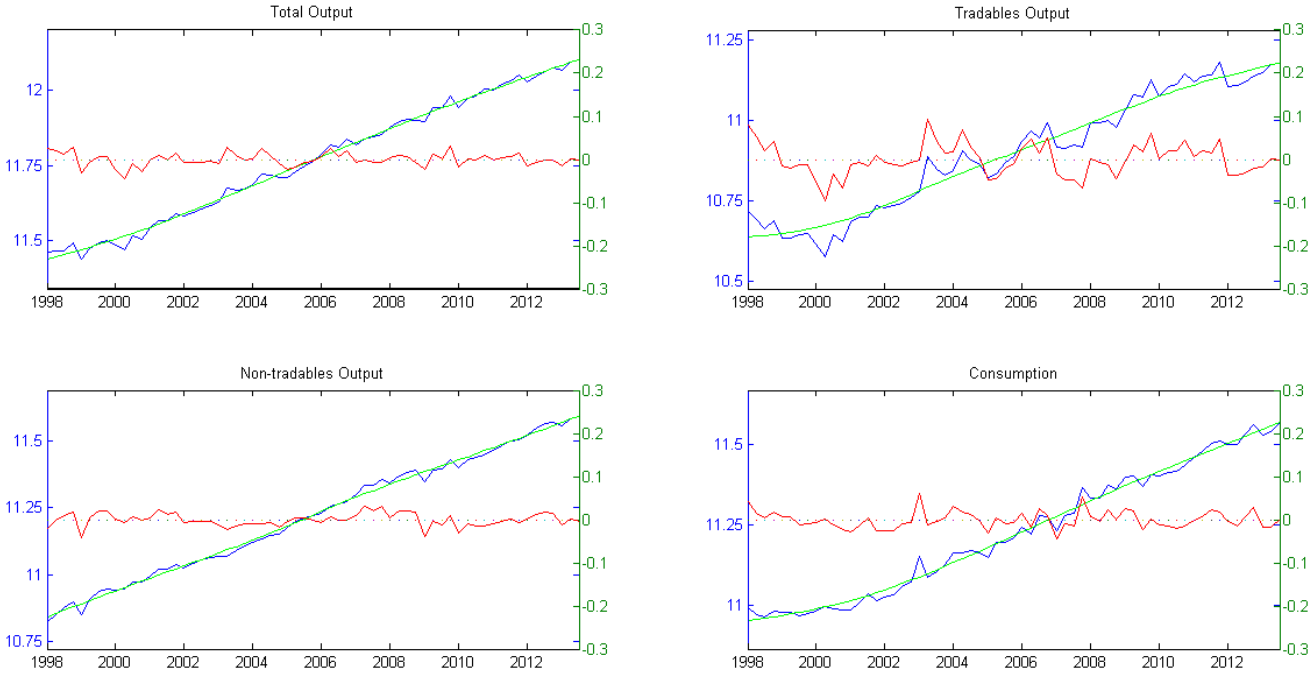
Some stylized facts of the Moroccan economy

Since the onset of the financial crisis of 2008, developing economies especially in the Middle East and North Africa (MENA) region were subject to several macroeconomic shocks: the global recession, strong fluctuations in commodity prices, the Euro area sovereign debt crisis. One must add to all of this the political turmoil following the Arab spring revolts. Compared to its neighbors, Morocco's economy showed a relative resilience or at least a slower reaction to those shocks due in part to its apparent political stability and its economic growth model.

During the 2000s, economic growth in Morocco was relatively high compared to other MENA oil importing countries. This growth was mainly attributable to a strong reliance on internal demand. The Moroccan government distributes generous indirect transfers to households through strongly subsidized prices for energy and basic agricultural commodities. Government plays also an important stabilizing role through public sector employment and investment expenditures. The informal sector contributed also by absorbing part of the shocks to employment and providing a social safety net for those excluded from the formal economy.

At the end of the 2000s, economic growth started to show signs of a slow-down. Public policies based on indiscriminate transfers to support consumption caused chronic budget deficits and failed to generate a private sector based growth able to absorb the ever growing unemployment among the educated stratas of the labor force. The effects of external shocks started to catch-up following the fall of the foreign demand and direct investments coming from the European Union and the reduction in the remittances on which the country relied so much for its foreign-exchange reserves.

FIGURE 3.1: Long-term trends and business cycle fluctuations around the trend (1)



In this chapter we discuss some stylized facts of the Moroccan economy. We use quarterly data for output, consumption, inflation, interest rates and other macroeconomic series from the period 1998-2013. We built the data series for tradable and non-tradable outputs using data for sectoral outputs. Following the usual practice, we consider agricultural and industrial outputs as tradables while services in general are considered as non-tradables. Imported inflation is computed using imports deflator while the data series for the terms of trade is computed using both import and export deflators.

3.1 Long-term trends

In what follows, we review some of the long-term trends of the Moroccan economy using quarterly data covering the period 1998-2013. Figures 3.1, 3.2 and 3.3 depict the long-term trends and the business cycle fluctuations around the trend for the main macroeconomic variables.

All real variables exhibit growth rates above 3%. The average annual growth rate over the covered period is 3.86% for GDP, 3.62% for consumption, 5.09% for investments, 3.82% for government spending. As it appears in figure 3.1, output growth speed increased substantially starting from the year 2000.

The average growth rate of consumption is lower than that of GDP due to a decline in the share of consumption. Indeed, the share of consumption in GDP fell down from 62.5% to 58.8%. The share of government spending stayed more or less around its long-term average of 16.38%.

In terms of volumes, exports and imports grew at the same pace with an average annual growth rate of 5.45% for exports and 5.33% for imports over the period 1998-2013. However, the real story is told through the terms of trade which declined steadily depicting a rise in the relative price of imports. This may be due to a loss in the competitiveness of the Moroccan economy. Exports fell drastically during the great recession but recovered quickly starting from the end of 2009. Imports decline was relatively slower in 2008 then imports stayed close to their long-term trend.

In line with the pegged exchange rate regime in place in Morocco, the nominal exchange rate remained constant during the covered period. The real exchange rate depreciated steadily during the same period. The continuous real depreciation didn't help exports take-off. This fact may imply that the bad performance of Moroccan exporting firms is more related to a loss of competitiveness rather than a strong national currency.

Over the long run, CPI inflation remained stable around 2% while fluctuating strongly in the short run. The period between 2005 and 2009 saw a significant increase in inflation due to the rise in imported inflation following commodity price movements in

FIGURE 3.2: Long-term trends and business cycle fluctuations around the trend (2)

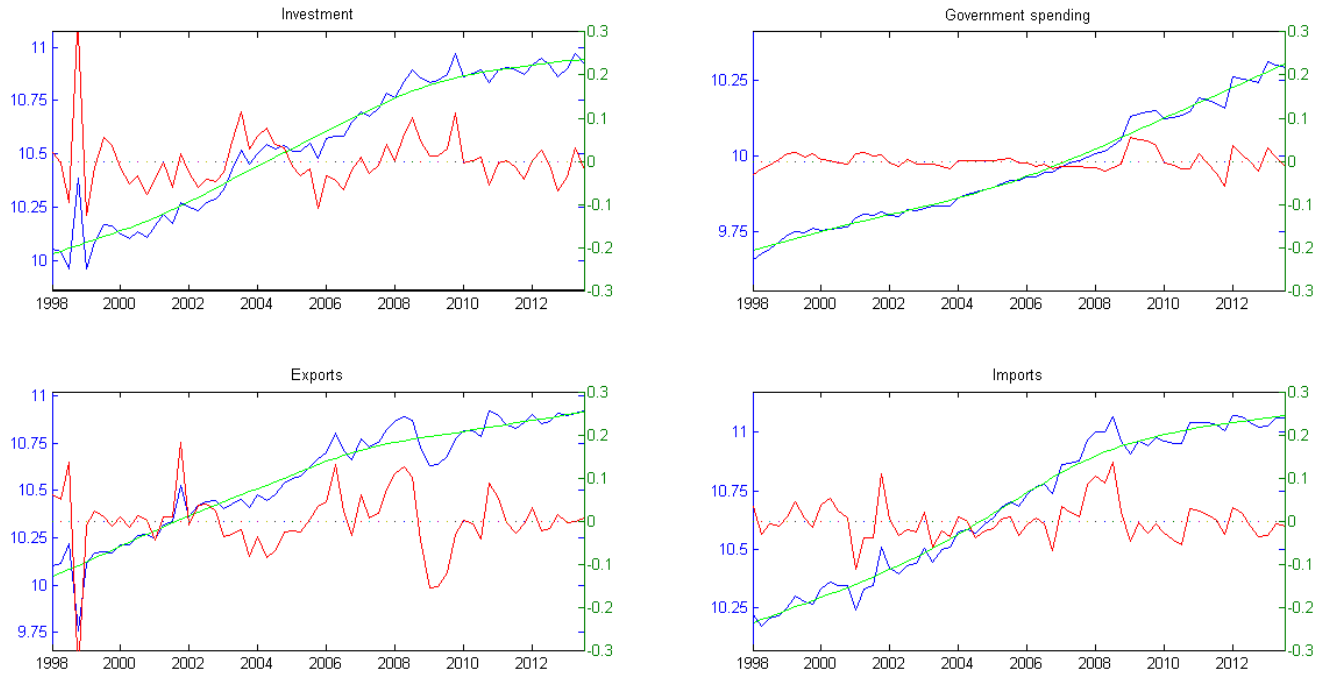
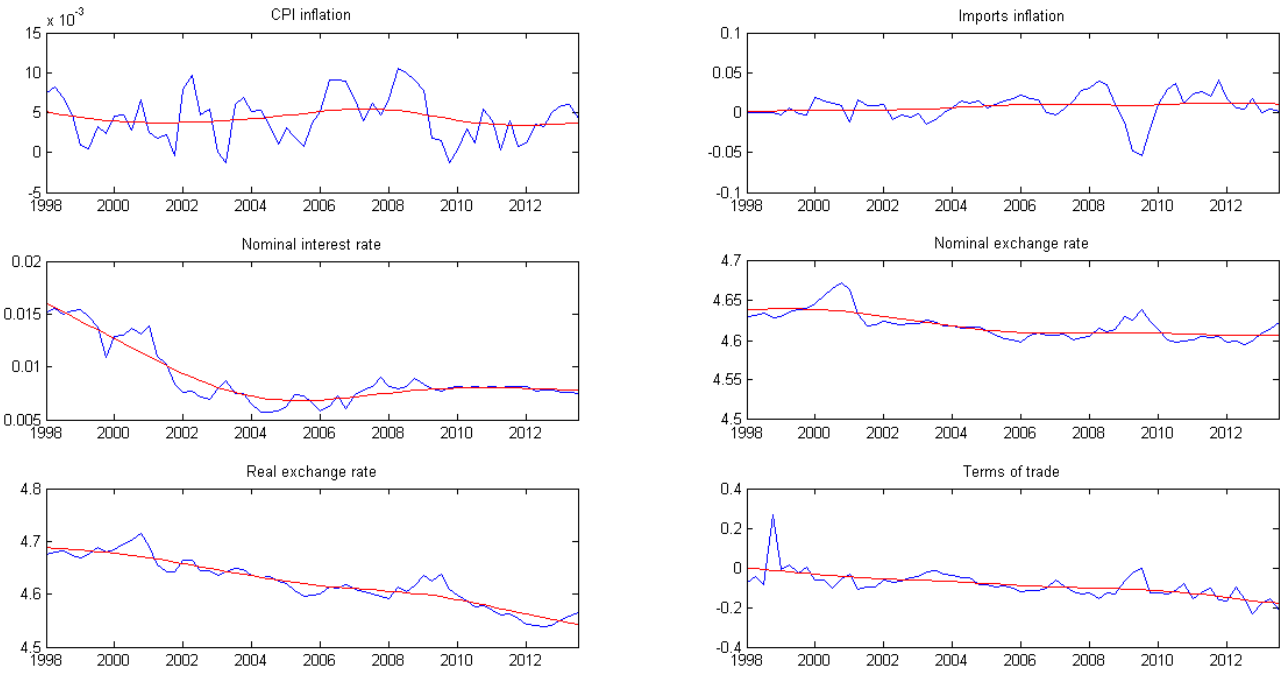


FIGURE 3.3: Long-term trends and business cycle fluctuations



international markets. A strong decline occurred in 2008 resulting from a fall in imported inflation caused by the global recession. This decline was followed by a stabilization around the 2% trend since 2010.

As we can see in figure 3.3, nominal interest rates followed a strong downward trajectory falling from 6% in 1998 to 2.3% in 2004. Starting from 2006, interest rates showed a mild increase to finally stabilize after 2009 around 3%.

3.2 Business cycle facts

Following the business cycles literature, we use HP-filtered quarterly data with a filtering parameter set at 1600. Table 3.1 presents some business cycle statistics computed over the period 1998-2013.

Output is less volatile compared to most other variables and its persistence is relatively low. Tradables output is significantly more volatile than non-tradables output which is consistent with the way the two series are computed. Consumption is more volatile than output in line with stylized facts of developing countries. Investment is highly volatile

TABLE 3.1: Business cycle statistics of Morocco

	Standard deviation	Relative standard deviation	1st order autocorrelation	Contemporaneous correlation with output
Y	1.51	1.00	0.20	1.00
Y_T	3.70	2.46	0.52	0.84
Y_N	1.51	1.00	0.29	0.21
C	2.02	1.34	0.15	0.20
I	6.37	4.23	-0.03	0.45
G	1.99	1.32	0.45	-0.22
X	7.61	5.06	0.21	-0.01
M	4.34	2.88	0.37	-0.09
π	0.28	0.18	0.49	0.02
R	0.09	0.06	0.59	-0.27
Q	1.52	1.01	0.71	-0.18
E	1.10	0.73	0.76	-0.24
S	5.08	3.38	0.08	0.16

Source: author calculations based on Morocco quarterly data (1998-2013)

and does not exhibit any persistence. Government expenditures are strongly counter-cyclical with a volatility close to output and consumption. This confirms the stabilizing role of government.

Imports are less volatile than exports since the last depend on external demand and economic conditions in international markets while Morocco's imports are composed mainly of agricultural commodities and energy products which are subject to a steady internal demand.

Nominal and real exchange rates volatility is in line with the volatility of inflation and output respectively. The strong persistence of exchange rates is due to the pegged exchange regime in place in Morocco as well as the strong restrictions on capital outflows.

The restrictions on capital mobility explain also the low volatility of nominal interest rates. The reason for the counter-cyclical character may be the primacy of inflation stabilization over output stabilization for the central bank. For example, in the case of a stagflation, the central bank will give priority to fighting inflation rather than boosting output. However, this explanation may not hold since nominal interest rates and inflation are negatively correlated.

Pro-cyclical macroeconomic variables include consumption, investment and the terms of trade. Counter-cyclical variables include government expenditures, interest rates and the real and nominal exchange rates. Exports, imports and inflation are acyclical.

Interest rates and real and nominal exchange rates are the most persistent variables while investment does not exhibit any persistence at all.

TABLE 3.2: Comparing business cycle statistics

	Morocco	Emerging countries	Developed countries
σ_Y	1.51	2.74	1.34
σ_C/σ_Y	1.34	1.46	0.94
σ_I/σ_Y	4.23	3.91	3.41
$\rho(Y_t, Y_{t-1})$	0.20	0.76	0.75
$\rho(C, Y)$	0.20	0.72	0.66
$\rho(I, Y)$	0.45	0.77	0.67

Source: Aguiar and Gopinath (2007) and author calculations based on Morocco quarterly data (1998-2013)

Table 3.2 presents a comparison between business cycle statistics in Morocco and average statistics in developed and developing countries as reported by Aguiar and Gopinath (2007).

The volatility of output in Morocco is closer to developed countries levels whereas the relative volatility of consumption and investment are closer to emerging countries levels. The relatively low volatility of output compared with other developing countries may be explained by the stabilizing interventions of the government, the small role of exports in the economy as well as the restrictions on capital mobility and the fixed exchange regime.

Output in Morocco exhibits lower persistence compared with both developed and developing economies. This might be related to a delayed counter-cyclical effect of government stabilization efforts.

The contemporaneous correlation with output of both consumption and investment are significantly lower than what is observed in both developing and developed countries.

It is worth noting that one should be careful with the relatively short period over which the business cycle statistics for Morocco were computed (1998-2013) as well as the possible discrepancy with the period over which Aguiar and Gopinath (2007) computed their statistics.

3.3 The informal economy in Morocco

Usually, the informal sector is defined as all economic activities that are not taxed nor subject to government monitoring. As a consequence, informal activities are not included

in national accounting statistics such as GDP and need to be measured through periodic surveys among the population.

The last official survey of the informal economy in Morocco was conducted in 2007 and the results were published in 2009 (HCP, 2009). According to this survey, the share of the informal sector in output was estimated at 14.3% in 2007 compared to 16.3% in 1999. The proportion of households living from informal activities went down from 18.2% in 1999 to 14.3% in 2007. The share of informal labor in non-agricultural labor was 37.3% against 39% in 1999.

The significant share of the informal sector in the Moroccan economy is a consequence of the historical movements of rural migration and uncontrolled urbanization starting from the 1970s (Mejjati Alami, 2006). This situation resulted in a rise of unemployment and poverty due to a failure of public policies to contain it. The formal economy with its limited growth model based mainly on imports substitution was unable to absorb the massive influx of labor force moving from rural areas, and its predominantly agricultural occupations, to urban areas. Informal activities filled the gap and played a social role of stabilization as an alternative to the failures of the formal economy and the welfare state (Mejjati Alami, 2006).

The majority of the informal production units and labor force is concentrated in the service sector, respectively 77.4% and 72.2% in 2007 (HCP, 2009)). This comforts our choice of modeling the informal sector as a sub-sector of the non-tradables sector as we will discuss later.

Informal production units are often confronted to problems such as the limited access to financing, to markets and to information. Only 1.1% of the informal production units make use of bank loans to finance their activities (HCP, 2009). Links with the formal economy such as subcontracting are very limited since only a small part of informal production units operate in the industrial sector (Mejjati Alami, 2006).

Chapter 4

The theoretical model

The model presented below tries to capture the basic structure of the Moroccan economy. Morocco is a relatively open economy which imports a large part of its needs especially in energy and agricultural commodities. For this reason, we adopt a small open economy framework that we integrate into a New-Keynesian model with nominal rigidities to account for the role of monetary policy. The theoretical model follows in a large part Gali and Monacelli (2005), Monacelli (2005) and Santacreu (2005).

The introduction of an informal sector both in labor and production into the model is justified by its significant place in Morocco economy as showed above. The informal labor sector is modeled as a perfectly competitive market providing labor to informal non-tradable sector firms. As in Ahmed et al. (2012), informal sector firms are labor intensive and use low productivity technology.

On the other hand, the formal labor market is shared between firms producing domestic tradable goods and firms producing formal non-tradable goods. This common labor market gives place to an interesting transmission mechanism between the two sectors as we will see later.

First we develop the model with a flexible exchange regime in mind and then try to introduce a fixed exchange regime in line with the current system in place in Morocco. The modeling of the foreign exchange rate regime and the monetary policy function are discussed at the end of this chapter.

4.1 Households

The domestic economy is populated by a representative household who derives its utility from consumption, C_t, and leisure, $1 - N_t$.

The representative household maximizes the following inter-temporal utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(\tilde{C}_t, N_t) \tag{4.1}$$

where $\beta \in [0,1]$ is the inter-temporal discount factor related to time preferences, E is the expectation operator and N_t is the total number of hours worked in the economy.

In equation (4.1), we have:

$$\tilde{C}_t \equiv C_t - hC_{t-1} \tag{4.2}$$

where h is the parameter of habit persistence and C_t is a constant elasticity of substitution (CES) consumption index defined by:

$$C_t = \left((1-\lambda)^{\frac{1}{\nu}} C_{T,t}^{\frac{\nu-1}{\nu}} + \lambda^{\frac{1}{\nu}} C_{N,t}^{\frac{\nu-1}{\nu}} \right)^{\frac{\nu}{\nu-1}} \tag{4.3}$$

where λ is the share of non-tradable goods and ν is the intratemporal elasticity of subsitution between tradable and non-tradable goods in the domestic economy.

The optimal allocation of any given expenditure between tradable and non-tradable goods yields the following demand functions:

$$C_{T,t} = (1-\lambda) \left(\frac{P_{T,t}}{P_t} \right)^{-\nu} C_t \tag{4.4}$$

$$C_{N,t} = \lambda \left(\frac{P_{N,t}}{P_t} \right)^{-\nu} C_t \tag{4.5}$$

where the CPI, the tradable goods price index and the non-tradable goods price index are defined as follows

$$P_t = \left[(1-\lambda) {P_{T,t}}^{1-\nu} + \lambda {P_{N,t}}^{1-\nu} \right]^{\frac{1}{1-\nu}} \tag{4.6}$$

$$P_{T,t} = \left[(1-\alpha) {P_{T,t}^H}^{1-\eta} + \alpha {P_{T,t}^F}^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{4.7}$$

$$P_{N,t} = \left[(1-\omega)(P_{N,t}^F)^{1-\mu} + \omega (P_{N,t}^I)^{1-\mu} \right]^{\frac{1}{1-\mu}} \tag{4.8}$$

Consumption of tradable goods is given by a composite consumption index defined by:

$$C_{T,t} = \left((1-\alpha)^{\frac{1}{\eta}} (C_{T,t}^H)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{T,t}^F)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{4.9}$$

Parameter $\eta > 0$ measures the elasticity of substitution between tradable domestic and imported goods. $\alpha \in [0,1]$ measures the share of imports consumption in the

consumption of tradables. $C^H_{T,t}$ represents a CES index of domestic tradable consumption goods given by:

$$C^H_{T,t} \equiv \left(\int_0^1 C^H_{T,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \, \mathrm{d}j \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{4.10}$$

where $j \in [0,1]$ denotes the goods variety, $\varepsilon > 1$ measures the elasticity of substitution between varieties (in any given country).

$C^F_{T,t}$ is a CES index of imported consumption goods given by:

$$C^F_{T,t} \equiv \left(\int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}} \, \mathrm{d}i \right)^{\frac{\gamma}{\gamma-1}} \tag{4.11}$$

where parameter $\gamma > 0$ measures the degree of substitution between goods produced in different foreign countries and $C_{i,t}$ is an index of the goods imported from the foreign country i given by:

$$C_{i,t} \equiv \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \, \mathrm{d}j \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{4.12}$$

The optimal allocation of expenditures between domestic tradables implies:

$$C^H_{T,t}(j) = \left(\frac{P^H_{T,t}(j)}{P^H_{T,t}} \right)^{-\varepsilon} C^H_{T,t} \tag{4.13}$$

where $P^H_{T,t}$ is the price index of domestic tradables expressed as:

$$P^H_{T,t} \equiv \left(\int_0^1 P^H_{T,t}(j)^{1-\varepsilon} \, \mathrm{d}j \right)^{\frac{1}{1-\varepsilon}} \tag{4.14}$$

In parallel, the optimal allocation of expenditures on imports by country of origin results in:

$$C_{i,t} = \left(\frac{P_{i,t}}{P^F_{T,t}} \right)^{-\gamma} C^F_{T,t} \tag{4.15}$$

where $P_{i,t}$ is the price index in domestic currency for imports from country i defined as:

$$P_{i,t} \equiv \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon} \, \mathrm{d}j \right)^{\frac{1}{1-\varepsilon}} \tag{4.16}$$

and $P^F_{T,t}$ is the price index in domestic currency for imported goods expressed as:

$$P^F_{T,t} \equiv \left(\int_0^1 P_{i,t}^{1-\gamma} \, \mathrm{d}i \right)^{\frac{1}{1-\gamma}} \tag{4.17}$$

Using the expressions above, the optimal allocation of goods between domestic tradables and imported goods yields the following demand functions (cf. Appendix C):

$$C_{T,t}^{H} = (1-\alpha)\left(\frac{P_{T,t}^{H}}{P_{T,t}}\right)^{-\eta} C_{T,t} \tag{4.18}$$

$$C_{T,t}^{F} = \alpha\left(\frac{P_{T,t}^{F}}{P_{T,t}}\right)^{-\eta} C_{T,t} \tag{4.19}$$

Consumption of non-tradable goods is allocated between goods produced in the formal sector and goods produced in the informal sector. This consumption bundle is given by a CES index composed by non-tradable formal sector goods, $C_{N,t}^{F}$, and informal sector goods, $C_{N,t}^{I}$, as follows:

$$C_{N,t} = \left((1-\omega)^{\frac{1}{\mu}}(C_{N,t}^{F})^{\frac{\mu-1}{\mu}} + \omega^{\frac{1}{\mu}}(C_{N,t}^{I})^{\frac{\mu-1}{\mu}}\right)^{\frac{\mu}{\mu-1}} \tag{4.20}$$

where ω is the share of informal consumption in non-tradables consumption and μ the elasticity of substitution between non-tradable formal and informal goods consumption. The larger the value of μ the higher the substitutability between the two goods categories.

As seen before, the household's minimization problem in the non-tradables consumption yields the following demand functions for the non-tradable formal and informal goods (cf. appendix B):

$$C_{N,t}^{F} = (1-\omega)\left(\frac{P_{N,t}^{F}}{P_{N,t}}\right)^{-\mu} C_{N,t} \tag{4.21}$$

$$C_{N,t}^{I} = \omega\left(\frac{P_{N,t}^{I}}{P_{N,t}}\right)^{-\mu} C_{N,t} \tag{4.22}$$

Total consumption expenditures by domestic households is given by:

$$P_{T,t}^{F}C_{T,t}^{F} + P_{T,t}^{H}C_{T,t}^{H} + P_{N,t}^{F}C_{N,t}^{F} + P_{N,t}^{I}C_{N,t}^{I} = P_t C_t \tag{4.23}$$

Assuming that households have access to complete international asset markets, the period budget constraint can be written as:

$$P_t C_t + \mathrm{E}_t[Q_{t,t+1}D_{t+1}] \leq D_t + W_t N_t + \Pi_t \tag{4.24}$$

where D_{t+1} is the nominal payoff in period $t+1$ of the portfolio held at the end of period t, $Q_{t,t+1}$ is the stochastic discount factor of the domestic household for one

period ahead nominal pay-offs, W_t is the nominal wage and Π_t the redistributed profits of the economy's firms.

We consider the following form for the utility function:

$$U(C,N) \equiv \frac{\tilde{C}^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi} \tag{4.25}$$

where σ is the inverse of the elasticity of substitution between consumption and labor and φ is the inverse labor elasticity.

The household maximizes his lifetime utility:

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\tilde{C}_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \tag{4.26}$$

subject to the intertemporal budget constraint (4.24).

Solving the household's utility maximization problem, one gets the usual optimality conditions:

$$\frac{N_t^{\varphi}}{\tilde{C}_t^{-\sigma}} = \frac{W_t}{P_t} \tag{4.27}$$

$$\beta R_t \mathrm{E}_t \left[\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right] = 1 \tag{4.28}$$

Section B.1 in appendix B presents the detailed derivation and the log-linearized optimality conditions.

4.2 Rest of the World

In what follows, the rest of the world is considered as a closed economy where the goods domestically produced represent a negligible fraction of the world's consumption. This implies that domestic and CPI inflation are equivalent in the rest of the world's economy.

In the rest of the world's economy, a representative consumer faces the same optimization problem as the domestic consumer and as a solution one obtains the same set of optimality conditions.

4.2.1 Bilateral terms of trade

The bilateral terms of trade are assumed to be exogenous to the small open economy. They are defined as the price of country i's goods in terms of home goods:

$$S_{i,t} = \frac{P_{i,t}}{P^H_{T,t}}$$

The effective terms of trade are defined as:

$$S_t \equiv \frac{P^F_{T,t}}{P^H_{T,t}} = \left(\int_0^1 S_{i,t}^{1-\gamma} \, \mathrm{d}i \right)^{\frac{1}{1-\gamma}} \tag{4.29}$$

In log, one gets:

$$s_t = p^F_{T,t} - p^H_{T,t}$$

A log-linearization of the effective terms of trade (first order approximation) yields:

$$s_t = \int_0^1 s_{i,t} \, \mathrm{d}i$$

Changes in the terms of trade represent changes in the competitiveness of the domestic economy.

4.2.2 CPI and domestic inflation

We define CPI, tradable goods and non-tradable goods inflation as:

$$\pi_t \equiv p_t - p_{t-1} \tag{4.30}$$

$$\pi_{T,t} \equiv p_{T,t} - p_{T,t-1} \tag{4.31}$$

$$\pi_{N,t} \equiv p_{N,t} - p_{N,t-1} \tag{4.32}$$

where $p \equiv \log P$.

The log-linearized CPI, tradable goods and non-tradable goods inflation are given by:

$$\pi_t = (1-\lambda)\pi_{T,t} + \lambda \pi_{N,t} \tag{4.33}$$

$$\pi_{T,t} = (1-\alpha)\pi^H_{T,t} + \alpha \pi^F_{T,t} \tag{4.34}$$

$$\pi_{N,t} = (1-\omega)\pi^F_{N,t} + \omega \pi^I_{N,t} \tag{4.35}$$

Domestic inflation is defined as a weighted average of domestic tradable goods inflation and non-tradable goods inflation:

$$\pi_t^d \equiv (1-\lambda)\pi_{T,t}^H + \lambda\pi_{N,t} \tag{4.36}$$

Log-linearizing the tradables price index expression in (4.7) around the symmetric steady state and combining it with the log expression for s_t, one gets:

$$p_{T,t} = p_{T,t}^H + \alpha s_t$$

Using the expressions above combined with the definitions of $\pi_{T,t}$ and $\pi_{T,t}^H$ one can write:

$$\pi_{T,t} = \pi_{T,t}^H + \alpha\Delta s_t$$

Replacing the above result in (4.33) one gets:

$$\pi_t = (1-\lambda)\pi_{T,t}^H + \lambda\pi_{N,t} + (1-\lambda)\alpha\Delta s_t$$

Using the expression for domestic inflation in (4.36) one finally gets:

$$\pi_t = \pi_t^d + (1-\lambda)\alpha\Delta s_t \tag{4.37}$$

According to this expression, CPI and domestic inflation are linked, with the gap between the two measures of inflation being proportional to the log change in the terms of trade. The coefficient of proportionality is given by an index of openness represented by the share of imports in households' overall consumption, $(1-\lambda)\alpha$.

4.2.3 Terms of trade and the nominal exchange rate

Following Gali and Monacelli (2005), we assume that the law of one price holds for import and export goods at all times which implies a complete pass-through of the exchange rate. We also assume that for the world countries continuum, there is no distinction between CPI and domestic price levels. This means that the prices of country i's goods can be written as:

$$P_{i,t}(j) = \mathcal{E}_{i,t}P_{i,t}^i(j) \qquad \forall i,j \in [0,1] \tag{4.38}$$

where the bilateral exchange rate $\mathcal{E}_{i,t}$ is the price of country i's currency in terms of the domestic currency and $P_{i,t}^i(j)$ is the price of country i's good in its own currency.

We use the previous expression in the definition of $P_{i,t}$ to get:

$$P_{i,t} = \mathcal{E}_{i,t} P^i_{i,t} \tag{4.39}$$

where $P^i_{i,t} \equiv \left(\int_0^1 P^i_{i,t}(j)^{1-\varepsilon} \, \mathrm{d}j \right)^{\frac{1}{1-\varepsilon}}$ is the price index of country i's goods expressed in its own currency.

We substitute into the expression for $P^F_{T,t}$ to get:

$$P^F_{T,t} = \left(\int_0^1 \left(\mathcal{E}_{i,t} P^i_{i,t} \right)^{1-\gamma} \, \mathrm{d}i \right)^{\frac{1}{1-\gamma}} \tag{4.40}$$

Log-linearizing the preceding expression around the symmetric steady state, one gets:

$$\begin{aligned} p^F_{T,t} &= \int_0^1 \left(e_{i,t} + p^i_{i,t} \right) \, \mathrm{d}i \\ &= e_t + p^*_t \end{aligned}$$

where $e_t \equiv \int_0^1 e_{i,t} \, \mathrm{d}i$ is the log nominal effective exchange rate and $p^*_t = \int_0^1 p^i_{i,t} \, \mathrm{d}i$ is log world price index.

Combining the preceding result with the expression for the terms of trade, $s_t = p^F_{T,t} - p^H_{T,t}$, one gets:

$$s_t = e_t + p^*_t - p^H_{T,t} \tag{4.41}$$

4.2.4 Terms of trade and the real exchange rate

We define the bilateral real exchange rate between the domestic economy and country i's economy as the ratio of both economies CPI expressed in the domestic currency:

$$\mathcal{Q}_{i,t} \equiv \frac{\mathcal{E}^i_t P^i_t}{P_t} \tag{4.42}$$

From that, one can derive the following expression:

$$\begin{aligned} q_t &= \int_0^1 \left(e_{i,t} + p^i_t - p_t \right) \, \mathrm{d}i \\ &= \int_0^1 e_{i,t} \, \mathrm{d}i + \int_0^1 p^i_t \, \mathrm{d}i - p_t \\ &= e_t + p^*_t - p_t \\ &= s_t + p^H_{T,t} - p_t \end{aligned}$$

Using $p^H_{T,t} = p_{T,t} - \alpha s_t$ in the last equality, one can write:

$$\begin{aligned} q_t &= s_t + p_{T,t} - \alpha s_t - p_t \\ &= (1-\alpha)s_t + p_{T,t} - ((1-\lambda)p_{T,t} + \lambda p_{N,t}) \\ &= (1-\alpha)s_t + \lambda(p_{N,t} - p_{T,t}) \end{aligned}$$

4.2.5 International risk sharing

Following Gali and Monacelli (2005), we assume perfectly integrated international bond markets and the same habit formation parameter in all countries. It follows that a first order condition symmetric to (4.28) must hold in any other country i:

$$\beta R_t \mathrm{E}_t \left[\left(\frac{\tilde{C}^i_{t+1}}{\tilde{C}^i_t} \right)^{-\sigma} \left(\frac{P^i_t}{P^i_{t+1}} \right) \left(\frac{\mathcal{E}^i_t}{\mathcal{E}^i_{t+1}} \right) \right] = 1 \tag{4.43}$$

where $R_t = \frac{1}{\mathrm{E}_t[Q_{t,t+1}]}$ is the one-period gross return on the risk-free bond denominated in the domestic currency. This expression means that the return on the risk-free bond must be the same in the domestic and foreign economy when expressed in the same currency.

Combining (4.28) and (4.43), one gets:

$$\mathrm{E}_t \left[\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right] = \mathrm{E}_t \left[\left(\frac{\tilde{C}^i_{t+1}}{\tilde{C}^i_t} \right)^{-\sigma} \left(\frac{P^i_t}{P^i_{t+1}} \right) \left(\frac{\mathcal{E}^i_t}{\mathcal{E}^i_{t+1}} \right) \right]$$

Using the definition of the real exchange rate $\mathcal{Q}_{i,t} \equiv \frac{\mathcal{E}^i_t P^i_t}{P_t}$, one can write:

$$\mathrm{E}_t \left[\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\sigma} \right] = \mathrm{E}_t \left[\left(\frac{\tilde{C}^i_{t+1}}{\tilde{C}^i_t} \right)^{-\sigma} \frac{\mathcal{Q}_{i,t}}{\mathcal{Q}_{i,t+1}} \right] \tag{4.44}$$

which can be rewritten as:

$$\tilde{C}_t = \Xi_i \tilde{C}^i_t \mathcal{Q}^{\frac{1}{\sigma}}_{i,t} \tag{4.45}$$

where $\Xi_i = \mathrm{E}_t \left[\left(\frac{\tilde{C}_{t+1}}{\tilde{C}^i_{t+1}} \right) \mathcal{Q}^{\frac{-1}{\sigma}}_{i,t+1} \right]$ is a constant which depends on initial relative net asset positions.

Assuming symmetric initial conditions with zero net foreign asset positions implies $\Xi_i = \Xi = 1$ and:

$$\tilde{C}_t = \tilde{C}^i_t \mathcal{Q}^{\frac{1}{\sigma}}_{i,t}$$

Taking logs on both sides, using (B.1.11) and integrating over i, one gets:

$$c_t - hc_{t-1} = c_t^* - hc_{t-1}^* + \frac{1-h}{\sigma} q_t \tag{4.46}$$

where c_t^* is the log index for the world consumption.

This equality describes a relationship linking domestic consumption, world consumption and the real exchange rate.

Next, we move forward the above expression one period and we combine it with the Euler equation from (B.1.13) to get:

$$c_t = \frac{h}{1+h} c_{t-1} + \frac{1}{1+h} \mathrm{E}_t[hc_t + c_{t+1}^* - hc_t^* + \frac{1-h}{\sigma} q_{t+1}] - \frac{1-h}{\sigma(1+h)} (r_t - \mathrm{E}_t[\pi_{t+1}]) \tag{4.47}$$

4.2.6 Uncovered Interest Parity

Assuming complete international financial markets, the price in terms of domestic currency of a risk-free bond denominated in foreign currency is given by:

$$\frac{\mathcal{E}_t^i}{R_t^i} = \mathrm{E}_t \left[Q_{t,t+1} \mathcal{E}_{t+1}^i \right] \tag{4.48}$$

Combining this equation with its domestic counterpart, $\mathrm{E}_t[Q_{t,t+1}] = \frac{1}{R_t}$, one gets the following uncovered interest rate parity (UIP) expression:

$$\mathrm{E}_t \left[Q_{t,t+1} \left(R_t - R_t^i \left(\frac{\mathcal{E}_{t+1}^i}{\mathcal{E}_t^i} \right) \right) \right] = 0 \tag{4.49}$$

Assuming that UIP holds means that investors are indifferent between investing in domestic or foreign assets.

Log-linearizing around the steady state and aggregating over i, one gets:

$$r_t - r_t^* = \mathrm{E}_t \left[\Delta e_{t+1} \right] + \varepsilon_{uip,t} \tag{4.50}$$

where $\varepsilon_{uip,t}$ is a shock parameter which captures deviations from UIP.

This expression shows that movements in the nominal exchange rate are determined by the wedge between domestic and foreign nominal interest rates.

4.3 Tradable sector firms

4.3.1 Cost minimization problem

We consider a continuum of monopolistic competitive firms using a Constant Return to Scale (CRS) technology. A typical firm produces a differentiated good using the following production function:

$$Y_{T,t}(j) = A_{T,t} N_{T,t}(j) \tag{4.51}$$

where $j \in [0,1]$ is an index of domestic tradable sector firms. This specification assumes that labor is the only factor of production with $A_{T,t}$ being the level of aggregate productivity.

In log-linearized terms:

$$y_{T,t}(j) = a_{T,t} + n_{T,t}(j) \tag{4.52}$$

where productivity, $a_{T,t}$, follows an AR(1) process

$$a_{T,t} = \rho_{a_{T,t}} a_{T,t-1} + \varepsilon_{a_{T,t}} \tag{4.53}$$

where $\varepsilon_{a_{T,t}} \sim \mathcal{N}(O, \sigma_{a_T})$.

We define $Y_{T,t} \equiv \left[\int_0^1 Y_{T,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \, \mathrm{d}j\right]^{\frac{\varepsilon}{\varepsilon-1}}$ as the index for domestic output and $N_{T,t} \equiv \int_0^1 N_{T,t}(j) \, \mathrm{d}j$ as the aggregate level of employment in the tradable goods sector. Using the previous definitions, one can define the aggregate domestic tradables output in log terms up to a first order approximation as:

$$y_{T,t} = a_{T,t} + n_{T,t} \tag{4.54}$$

The typical tradable sector firm tries to minimize production costs and maximize profits. In the first step, the firm minimizes costs by choosing the lowest possible level of labor:

$$\min \frac{W_t^F}{P_{T,t}^H} N_{T,t}(j) \tag{4.55}$$

subject to the production function in (4.51).

This optimization problem results in:

$$\frac{W_t^F}{P_{T,t}^H A_{T,t}} \equiv MC_{T,t}$$

where $MC_{T,t}$ denotes the real marginal cost which equals the real wage divided by the marginal productivity of labor, $A_{T,t}$. Marginal cost is increasing in nominal wage and decreasing in prices and labor marginal productivity.

4.3.2 Price Setting

In the second step, firms maximize profits, generated through the sale of production $Y_{T,t}(j)$ minus production costs $\lambda_t Y_{T,t}(j)$, by setting their prices $P_{T,t}(j)$ subject to the demand curve for their goods given by (4.18) and the assumption of sticky prices.

Following Gali and Monacelli (2005), we assume that domestic tradable sector firms set prices in a staggered fashion à la Calvo (1983).

Each period, a measure $1-\theta_T$ of (randomly selected) firms set optimally new prices. The remaining θ_T firms keep their prices unchanged. The average duration of a price is given by $\frac{1}{1-\theta_T}$. The probability that a firm re-optimizes its price is independent of the time elapsed since the last price adjustment. The parameter θ is a measure of the degree of nominal rigidity i.e. the larger θ the fewer the firms that adjust their prices each period and the longer the expected time between price changes.

Let $\bar{P}^H_{T,t}(j)$ be the price set by a firm adjusting its price in period t. Under the Calvo price setting, $P^H_{T,t+k}(j) = \bar{P}^H_{T,t}(j)$ with probability θ^k_T for $k = 0, 1, 2, \ldots$. For convenience, we drop the j subscript since all re-optimizing firms will choose the same price.

The profit maximization problem can be expressed as follows:

$$max_{\bar{P}^H_{T,t}} \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k_T \mathrm{E}_t \left[Q_{t,t+k} \left(Y_{T,t+k}(\bar{P}^H_{T,t} - MC^n_{T,t+k}) \right) \right] \tag{4.56}$$

subject to the assumption of Calvo pricing and the following demand function:

$$Y_{T,t+k}(j) = \left(\frac{\bar{P}^H_{T,t}(j)}{P^H_{T,t+k}} \right)^{-\varepsilon} \left(C^H_{T,t+k} + \int_0^1 C^{H,i}_{T,t+k} \, \mathrm{d}i \right) \tag{4.57}$$

where $MC^n_{T,t} = \frac{W_{T,t}}{A_{T,t}}$ is the nominal marginal cost.

After some mathematical derivations (cf. section B.2 in appendix B), one obtains the following New Keynesian Philips Curve for the tradables sector:

$$\pi^H_{T,t} = \beta \mathrm{E}_t \left[\pi^H_{T,t+1} \right] + \lambda_T \left(mc_{T,t} - mc_T \right) \tag{4.58}$$

where $\lambda_T = \frac{(1-\theta_T)(1-\beta\theta_T)}{\theta_T}$.

4.4 Non-tradable formal sector firms

4.4.1 Cost minimization problem

Similar to the tradable sector firms, the non-tradable formal sector firms produce a differentiated good using the following CRS production function:

$$Y^F_{N,t}(j) = A_{N,t} N^F_{N,t}(j) \tag{4.59}$$

where $j \in [0,1]$ is an index of non-tradable formal sector firms.

In log-linearized terms:

$$y^F_{N,t}(j) = a_{N,t} + n^F_{N,t}(j) \tag{4.60}$$

where productivity, $a_{N,t}$, follows an AR(1) process

$$a_{N,t} = \rho_{a_{N,t}} a_{N,t-1} + \varepsilon_{a_{N,t}} \tag{4.61}$$

where $\varepsilon_{a_{N,t}} \sim \mathcal{N}(O, \sigma_{a_N})$.

We define $Y^F_{N,t} \equiv \left[\int_0^1 Y^F_{N,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \,\mathrm{d}j\right]^{\frac{\varepsilon}{\varepsilon-1}}$ as the index for non-tradable formal output and $N^F_{N,t} \equiv \int_0^1 N^F_{N,t}(j)\,\mathrm{d}j$ the aggregate level of employment in the non-tradable formal goods sector.

As for the tradable sector firms, the typical non-tradable formal sector firm tries to minimize production costs and maximize profits. Solving the cost minimization problem one gets the expression for the marginal cost:

$$\frac{W^F_t}{P^F_{N,t} A_{N,t}} \equiv MC_{N,t}$$

where $MC_{N,t}$ denotes the real marginal cost which equals the real wage, $\frac{W^F_t}{P_{N,t}}$, divided by the marginal productivity of labor, $A_{N,t}$. Here again, the real marginal cost is increasing in nominal wage and decreasing in prices and labor marginal productivity.

4.4.2 Price setting

As with tradable sector firms, the second optimization problem facing non-tradable formal sector firms consists in maximizing profits given the results of the first step. Profits are the difference between income from sales $Y^F_{N,t}(j)$ and the production costs

$\lambda_t Y^F_{N,t}(j)$, by setting their prices $P^F_{N,t}(j)$ subject to the demand curve for their goods given by (4.18) and the assumption of sticky prices.

In parallel with the tradable sector firms, we assume Calvo (1983) price setting for the non-tradable formal sector firms. Each period, a measure $1-\theta_N$ of (randomly selected) firms set optimally new prices. Let $\bar{P}^F_{N,t}(j)$ be the price set by a firm adjusting its price in period t. Under the Calvo price setting, $P^F_{N,t+k}(j) = \bar{P}^F_{N,t}(j)$ with probability θ^k_N for $k = 0, 1, 2, \ldots$. Here again, we drop the j subscript since all re-optimizing firms will choose the same price.

As before, the profit maximization problem can be expressed as follows:

$$max_{\bar{P}^F_{N,t}} \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k_N \mathrm{E}_t \left[Q_{t,t+k} \left(Y^F_{N,t+k} (\bar{P}^F_{N,t} - MC^n_{N,t+k}) \right) \right] \tag{4.62}$$

subject to the assumption of Calvo pricing and the following demand function:

$$Y^F_{N,t+k}(j) = \left(\frac{\bar{P}^F_{N,t}(j)}{P^F_{N,t+k}} \right)^{-\varepsilon} \left(C^F_{N,t+k} \right) \tag{4.63}$$

where $MC^n_{N,t} = \frac{W^F_{N,t}}{A_{N,t}}$ is the nominal marginal cost.

Following the steps detailed in section B.2, one gets the following New Keynesian Philips Curve for the formal non-tradables sector:

$$\pi^F_{N,t} = \beta \mathrm{E}_t[\pi^F_{N,t+1}] + \lambda_N \left[mc_{N,t} - mc_N \right] \tag{4.64}$$

where $\lambda_N \equiv \frac{(1-\theta_N)(1-\beta\theta_N)}{\theta_N}$

4.5 Non-tradable informal sector firms

Informal sector firms operate in a perfectly competitive market. The production function is given by:

$$Y^I_{N,t}(j) = \zeta N^I_{N,t}(j) \tag{4.65}$$

Informal sector firms solve the following profit maximization problem:

$$\max \; Y^I_{N,t} - \frac{W^I_{N,t}}{P^I_{N,t}} N^I_{N,t} \tag{4.66}$$

subject to the production function in (4.65) which yields the following expression:

$$P^I_{N,t} = \frac{W^I_{N,t}}{\zeta} \tag{4.67}$$

4.6 Household labor supply decision

Household's aggregate supply of labor, N_t, is a composite of both formal, N^F_t, and informal, N^I_t, labor. A fraction κ of households provides labor to the formal firms operating in the tradable and the formal non-tradable sectors and the rest, $(1-\kappa)$, provides labor to the informal firms. Aggregate labor supply is given by the following CES index:

$$N_t = \left[\kappa^{-\tau}(N^F_t)^{1+\tau} + (1-\kappa)^{-\tau}(N^I_t)^{1+\tau}\right]^{\frac{1}{1+\tau}} \tag{4.68}$$

In the same manner, aggregate wage can be represented as follows:

$$W_t = \left[\kappa(W^F_t)^{\frac{\tau}{1+\tau}} + (1-\kappa)(W^I_{N,t})^{\frac{\tau}{1+\tau}}\right]^{\frac{1+\tau}{\tau}} \tag{4.69}$$

where τ is the inverse elasticity of substitution between formal and informal labor. W^F_t and W^I_t represent nominal wages in the formal (tradable and non-tradable) and informal sectors respectively.

We assume labor mobility between tradables and formal non-tradables sectors in the following way:

$$N^F_t = N_{T,t} + N^F_{N,t} \tag{4.70}$$

This allows for the enforcement of the same level of wages in both formal sectors. We also assume that the steady state share of both sectors in formal labor, χ, is the same.

The representative household optimizes his aggregate wage by choosing the amount of work allocated to the formal and informal sectors. This optimization problem yields the following supply functions for non-tradable formal and informal labor:

$$N^F_t = \kappa\left(\frac{W^F_t}{W_t}\right)^{\frac{1}{\tau}} N_t \tag{4.71}$$

$$N^I_t = (1-\kappa)\left(\frac{W^I_t}{W_t}\right)^{\frac{1}{\tau}} N_t \tag{4.72}$$

Both labor supply equations tell us that supply for each type of labor depends on its relative wage as well as the aggregate labor supply.

4.7 Monetary policy and the exchange rate regime

Morocco is classified by the IMF as having a conventional fixed exchange regime with a nominal exchange rate peg to a currency composite (IMF, 2013).

Under a combination of fixed exchange rate regime and free capital movements, the central bank loses its capacity to use interest rate policy for domestic stabilization purposes. This is because the central bank needs to stick strictly to the UIP in order to avoid destabilizing capital movements. In this case, assuming $\Delta e_t = 0$ implies the domestic interest rate is a function of the interest rate of the world economy:

$$r_t = r_t^* \tag{4.73}$$

In the case of Morocco, capital movements are restricted. This isolates to a large extent domestic interest rates from fluctuations in its foreign counterparts. In such a situation, the central bank enjoys some leverage over monetary policy.

After the revision of the legal status of the Central Bank of Morocco (Bank Al-Maghrib) in 2006, price stability became its principal objective (Bank Al-Maghrib, 2006). It was also given the ability to determine the appropriate exchange rate within the framework of an exchange rate peg to a currency composite. To pursue its objective, the Central Bank of Morocco monitors several inflation indicators and uses the overnight interest rate as its main policy tool (Bank Al-Maghrib, 2006).

To better reflect the monetary policy framework described above, we postulate an interest rate feedback rule where the central bank responds to deviations of output, inflation and changes in the real exchange rate:

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1-\rho_r)(\psi_y \hat{y}_t + \psi_\pi \pi_t + \psi_q \hat{q}_t) + \varepsilon_{r,t} \tag{4.74}$$

This rule does not imply that the central bank targets an explicit level of the real exchange rate but that large fluctuations in the real exchange rate may trigger a policy reaction from the central bank.

To account for the fixed exchange rate regime and the tight control on capital outflows, we replace the UIP equation in the model by the following equation auto-regressive process for the nominal exchange rate:

$$\hat{e}_t = \rho_e \hat{e}_{t-1} + \varepsilon_{e,t} \tag{4.75}$$

Replacing 4.50 by 4.75 isolates the local economy from world interest rate shocks and keeps the nominal exchange rate fixed. This choice is comforted by the overwhelming empirical evidence rejecting the UIP condition at least in the short run.

Following the same reasoning, we replace back the international risk sharing equation (4.47) by the standard Euler equation (B.1.13) typical of a closed economy.

4.8 General equilibrium

For each good variety j produced by the domestic tradable sector, the market clearing condition requires that production is equal to domestic and foreign consumption. This condition is given by:

$$\begin{aligned} Y_{T,t}(j) &= C^H_{T,t}(j) + \int_0^1 C^{H,i}_{T,t}(j)\,\mathrm{d}i \\ &= \left(\frac{P^H_{T,t}(j)}{P^H_{T,t}}\right)^{-\varepsilon} C^H_{T,t} + \int_0^1 C^{H,i}_{T,t}(j)\,\mathrm{d}i \end{aligned}$$

where $C^{H,i}_{T,t}(j)$ denotes demand from country i for good j produced by the domestic tradable sector. To find an expression for $C^{H,i}_{T,t}(j)$ we use equations (4.15), (4.19) and the assumption of symmetric preferences across countries to write:

$$C^{H,i}_{T,t}(j) = \alpha \left(\frac{P^H_{T,t}(j)}{P^H_{T,t}}\right)^{-\varepsilon} \left(\frac{P^H_{T,t}}{\mathcal{E}_{i,t}P^{F,i}_{i,t}}\right)^{-\gamma} \left(\frac{P^{F,i}_{i,t}}{P^i_{i,t}}\right)^{-\eta} C_{i,t} \tag{4.76}$$

Plugging back this expression into the expression for $Y_{T,t}(j)$, we can write:

$$Y_{T,t}(j) = \left(\frac{P^H_{T,t}(j)}{P^H_{T,t}}\right)^{-\varepsilon} \left[C^H_{T,t} + \alpha \int_0^1 \left(\frac{P^H_{T,t}}{\mathcal{E}_{i,t}P^{F,i}_{i,t}}\right)^{-\gamma} \left(\frac{P^{F,i}_{i,t}}{P^i_{i,t}}\right)^{-\eta} C_{i,t}\,\mathrm{d}i\right]$$

Using the definition for aggregate domestic tradable output $Y_{T,t} \equiv \left[\int_0^1 Y_{T,t}(j)^{\frac{\varepsilon-1}{\varepsilon}}\,\mathrm{d}j\right]^{\frac{\varepsilon}{\varepsilon-1}}$ one gets:

$$Y_{T,t} = C^H_{T,t} + \alpha \int_0^1 \left(\frac{P^H_{T,t}}{\mathcal{E}_{i,t}P^{F,i}_{i,t}}\right)^{-\gamma} \left(\frac{P^{F,i}_{i,t}}{P^i_{i,t}}\right)^{-\eta} C_{i,t}\,\mathrm{d}i$$

Since we assumed no distinction between CPI and domestic prices in each foreign country i, we have $\frac{P^{F,i}_{i,t}}{P^{i}_{i,t}} = 1$. Hence we can write:

$$\begin{aligned} Y_{T,t} &= C^{H}_{T,t} + \alpha \int_0^1 \left(\frac{P^{H}_{T,t}}{\mathcal{E}_{i,t} P^{i}_{i,t}} \right)^{-\gamma} C_{i,t} \, \mathrm{d}i \\ &= C^{H}_{T,t} + \alpha \int_0^1 \left(\frac{P^{H}_{T,t}}{P_t} \right)^{-\gamma} \left(\frac{P_t}{\mathcal{E}_{i,t} P^{i}_{i,t}} \right)^{-\gamma} C_{i,t} \, \mathrm{d}i \end{aligned}$$

Using the definition for the bilateral real exchange rate and integrating over i, we get the market clearing condition in the domestic domestic tradables goods sector:

$$Y_{T,t} = C^{H}_{T,t} + \alpha \left(\frac{P^{H}_{T,t}}{P_t} \right)^{-\gamma} \left(\frac{1}{\mathcal{Q}_t} \right)^{-\gamma} C^{*}_{t}$$

where the second term of the right hand side is the definition of exports:

$$C^{H,*}_{T} \equiv \alpha \left(\frac{P^{H}_{T,t}}{P_t} \right)^{-\gamma} \left(\frac{1}{\mathcal{Q}_t} \right)^{-\gamma} C^{*}_{t} \tag{4.77}$$

The market clearing condition in the non-tradable formal and informal sectors are given by:

$$Y^{F}_{N,t} = C^{F}_{N,t} \tag{4.78}$$

$$Y^{I}_{N,t} = C^{I}_{N,t} \tag{4.79}$$

Chapter 5

Calibration, estimation and main results

5.1 Model parametrization

For the purpose of calibrating and estimating the model, we use seasonally adjusted data from 1998 to 2013Q3. The data is log-scaled and detrended using HP-filter. Since the model is in log-deviations from the steady state, all the variables in the data are in the form of fluctuations around their dynamic trends.

We use a combination of calibration and estimation approaches to parameterize the model. More specifically, we distinguish between three groups of parameters:

1. **Steady state parameters**: describe the underlying structure of Morocco economy and are parameterized using long-term averages taken from data;
2. **Auto-regressive parameters**: coefficients of the exogenous AR stochastic processes are fitted from data using OLS.
3. **Behavioral parameters**: describe the deep structural parameters of Morocco economy. This group is estimated using Bayesian estimation approach with prior distributions based on the existing literature.

5.1.1 Calibration

Table 5.1 lists the steady state parameters. Those parameters are calibrated using the available data on the main macroeconomic variables and the informal sector.

TABLE 5.1: Steady state parameters and their calibrated values

Parameter	Description	Value
β	Rate of time preference (subjective discount factor)	0.99
λ	Share of non-tradables in aggregate consumption/output	0.58
ω	Share of informal consumption in the non-tradables consumption	0.25
α	Share of imports consumption in the consumption of tradables	0.80
κ	Fraction of households providing labor to formal firms	0.75
χ	Share of tradable labor in formal labor market	0.50
ι	Share of exports in tradable output	0.70

The share of non-tradables in output and consumption is assumed to be the same. This share is set at 58% based on the average ratio of non-tradable GDP to total GDP. The share of imports in the consumption of tradables is calibrated at 80% based on the average historical share of imports in tradable GDP. The share of exports in output is calibrated at 70% based on the average share of exports in tradable GDP.

The share of informal consumption in the non-tradables consumption is computed using the average share of non-tradables in output (58%) and the share of the informal sector estimated around 15% (14.3% according to the 2007 survey, (HCP, 2009)). The share of informal consumption in the non-tradables is then estimated around 25%.

According to the 2007 informal sector survey, (HCP, 2009), informal labor constitutes 37.3% of non-agricultural labor which in its turn is on average around 60% of total working population. Based on this information, we safely assume the fraction of households providing labor to formal sector firms to be around 75%. From this fraction, we assume that 50% are employed by firms operating in the tradable sector.

The rate of time preference, β, is fixed to the usual 0.99 which implies an annual steady state interest rate of 4% which is close to the average interest rate observed in the data.

TABLE 5.2: Auto-regressive parameters and their calibrated values

Parameter	Description	Value
ρ_e	Nominal exchange rate persistence	0.80
σ_e	Nominal exchange rate shock	0.01
ρ_{c^*}	World consumption persistence	0.89
ρ_{π^*}	World inflation persistence	0.76
ρ_{r^*}	World interest rate persistence	0.89
σ_{c^*}	World consumption shock	0.07
σ_{π^*}	World inflation shock	0.06
σ_{r^*}	World interest rate shock	0.06

The nominal exchange rate auto-regressive process is fitted using OLS based on quarterly data for Morocco. We do the same for the World economy auto-regressive processes using quarterly data for Euro area real GDP, year-on-year CPI inflation and 3 months money market interest rates covering the period 1998-2013. All Euro area data series are detrended using HP-filter. Table 5.2 shows the results.

5.1.2 Bayesian estimation

According to Lubik and Schorfheide (2006), a Bayesian estimation approach has three advantages over limited information approaches. First, Bayesian estimation is system-based and uses all the information provided by the data by fitting the DSGE model to a vector of aggregate time series. Second, the estimation is based on the likelihood function generated by the model rather than the minimization of a distance function between the model's moments, e.g. the impulse response functions, and empirical VAR corresponding moments. Third, prior distributions used in the Bayesian approach incorporates additional information not included in the data into parameters estimates. Prior distributions may reflect subjective judgment, conventional wisdom, evidence from previous studies or results from micro-level data.

5.1.2.1 Priors specification

The choice of priors for the Bayesian estimation of the model's behavioral parameters is based in part on empirical evidence from previous studies. However, estimated DSGE literature for emerging economies is very limited and is virtually inexistent for Morocco. In this case, one can at best look at similar developing economies or otherwise draw from the abundant DSGE literature for developed economies for parameters related to similar stylized facts.

Following the literature, we specify a Beta distribution for parameters constrained on the unit interval, a Gamma distribution for parameters in $\mathbb{R}_+$ and an inverse Gamma distribution for variance parameters. In particular, we use Beta distributions for the persistence parameters of the auto-regressive processes, Gamma distributions for the weights of the variables in the Taylor rule and inverse Gamma distributions for the standard deviations. Elasticity parameters are assumed to follow a normal distribution. Prior means are taken from the literature. We choose larger standard deviations of the prior distributions in order to cover a greater range of parameter values and give more freedom for the data to choose the parameters' location. Table 5.3 provides a list of the behavioral parameters and the priors used for their estimation.

For the habit persistence parameter h, we borrow the posterior values reported by Peiris and Saxegaard (2007) for their model of low income African countries by specifying a Beta distribution with a mean of 0.25 and a standard deviation of 0.1.

In line with the literature, we assume a normal distribution for σ, the inverse of the inter-temporal elasticity of substitution, with a mean of 3 and a standard deviation of 0.1.

Following Acosta et al. (2009), we assume a normal distribution with a mean of 0.4 for ν, the elasticity of substitution between tradable and non-tradable goods consumption, and a mean of 0.7 for η, the elasticity of substitution between tradable domestic and foreign goods. Acosta et al. (2009) justify this choice by claiming that domestic and foreign tradables have a relatively higher degree of substitution than do tradables with non-tradables. The standard deviations are set at 0.1.

Borrowing from Ahmed et al. (2013) for Pakistan's economy, we set the mean of the elasticity of substitution between non-tradable formal and informal consumption, μ, at 0.7 with a standard deviation of 0.1.

Following the calibrated value from Gali and Monacelli (2005), we set a Gaussian prior for the elasticity of substitution between foreign goods, γ, with a mean of 1 and a standard deviation of 0.1.

For the inverse labor elasticity, ϕ, we follow Fagan and Messina (2009) and Ahmed et al. (2013) and set a Gaussian prior with a mean of 1.5 and a standard deviation of 0.1. The mean value is consistent with the posterior reported by Smets and Wouters (2007).

The inverse of the elasticity of substitution between formal and informal labor, τ, is set to follow a normal distribution with a mean of 2 and a standard deviation of 0.5. Those values are close to the ones found by Ahmed et al. (2012) using labor force survey data from Pakistan.

The persistence parameters for the productivity processes are assumed to follow a Beta distribution with a mean set at 0.75 and a standard deviation set at 0.1.

Following Smets and Wouters (2003), we use inverted Gamma distributions with a degree of freedom equal to 2 for the variances of the shock processes. This distribution guarantees a positive variance. The means are set at 0.1.

Both Calvo parameters are assumed to follow a Beta distribution. These parameters indicate for each sector the fraction of firms unable to reset their prices at a certain period. The mean for both the domestic tradable, θ_t, and the formal non-tradable, θ_n, sectors is set at 0.75 which corresponds to an average fixed pricing period of 4 quarters.

The standard deviation is set at 0.05 to allow for variations between 3 quarters and 2 years (Smets and Wouters, 2003).

Following the work of Gabriel et al. (2010) on the Indian economy, priors for the parameters of the monetary policy reaction function are set in a loose manner in order to cover a large parameter domain. This choice reflects a lack of knowledge regarding the way monetary policy was conducted in Morocco before 2006. Feedback parameters have Gamma distributions with means set at 2 and standard deviations of 0.5. The lagged interest rate parameter follows a Beta distribution with mean 0.6, taken from the nominal interest rate auto-correlation as computed in business cycle statistics, and a standard deviation of 0.1. Setting loose prior distributions will allow us to investigate the effect of the informal sector on the reaction function and compare it with the standard values found in the literature.

We use Dynare (Adjemian et al., 2011) for the Bayesian estimation of our model. Bayesian estimation under Dynare needs at least as many shocks as there are observables to avoid stochastic singularity. We use the transformed series for tradable output, non-tradable formal output, CPI inflation, the nominal interest rate and the real exchange rate.

We experimented with other observables such as consumption, imports inflation and the terms of trade but the estimation process did not converge. In addition, total consumption as defined in the model does not correspond to the formal consumption reported in the national accounting data. Unfortunately, we know from Guerron-Quintana (2010) that the estimated parameter values are very sensitive to the data series used for the estimation.

The use of formal tradable and non-tradable output series instead of total output is justified by our decision of considering total output as an unobserved variable since it includes informal output whereas the available data for total output considers only formal activities. This choice is fully consistent with our theoretical model.

TABLE 5.3: Prior and posterior distributions for the behavioral parameters

Parameter	Description	Prior distribution			Posterior distribution		
		Density	Mean	Std. Dev.*	Mean	95% HPD interval	
h	Habit persistence parameter	Beta	0.25	0.10	0.06	0.01	0.11
σ	Inverse of the elasticity of substitution between consumption and labor	Normal	3.00	0.10	2.97	2.81	3.14
ν	Elasticity of substitution between tradable and non-tradable goods consumption	Normal	0.40	0.10	0.10	-0.09	0.28
η	Elasticity of substitution between domestic tradable and imported goods	Normal	0.70	0.10	0.61	0.45	0.78
μ	Elasticity of substitution between non-tradable formal and informal goods consumption	Normal	0.70	0.10	0.61	0.44	0.78
γ	Elasticity of substitution between goods produced in different foreign countries	Normal	1.00	0.10	1.17	0.99	1.34
ϕ	Inverse labor elasticity	Normal	1.50	0.10	1.57	1.40	1.73
τ	Inverse elasticity of substitution between formal and informal labor	Normal	2.00	0.50	1.85	1.02	2.70
ρ_{a_t}	Productivity persistence for domestic tradable firms	Beta	0.75	0.10	0.85	0.76	0.94
ρ_{a_n}	Productivity persistence for formal non-tradable firms	Beta	0.75	0.10	0.95	0.92	0.98
σ_{a_t}	Productivity shock for domestic tradable firms	Inv. Gamma	0.10	2.00	0.03	0.02	0.04
σ_{a_n}	Productivity shock for formal non-tradable firms	Inv. Gamma	0.10	2.00	0.04	0.04	0.05
θ_t	Calvo parameter for tradables firms	Beta	0.75	0.1	0.89	0.84	0.95
θ_n	Calvo parameter for non-tradables firms	Beta	0.75	0.1	0.26	0.16	0.35
ρ_r	Taylor rule interest rate persistence	Beta	0.60	0.10	0.51	0.37	0.64
ψ_y	Taylor rule output	Gamma	2.00	0.50	0.88	0.59	1.15
ψ_π	Taylor rule inflation	Gamma	2.00	0.50	5.77	4.88	6.83
ψ_q	Taylor rule real exchange rate	Gamma	2.00	0.50	0.94	0.67	1.21
σ_r	Monetary policy shock	Inv. Gamma	0.10	2.00	0.02	0.01	0.02

* For the inverted Gamma distributions, the degrees of freedom are indicated

5.1.2.2 Posterior estimates

Figure C.3 in appendix B presents the results of the identification tool provided by Dynare (Adjemian et al., 2011). It shows that all parameters are identified from the data used with the micro-behavioral parameters having the lowest identification strength (with the exception of σ). This can be improved upon by the addition of consumption and labor related data series.

The estimation results are reported in table 5.3. The last three columns report the posterior means along with the 95% Bayesian probability intervals based on the posterior probability densities. Figures C.1 and C.2 in appendix B provide a visual presentation of the estimation results by plotting together the prior distributions (grey line), the posterior distributions (black line) and the posterior modes (dashed green line).

In general, posterior distributions are reasonable and do not exhibit wild results. In cases when the posterior distribution is tighter than the prior distribution, data is very informative. For some parameters, the prior and posterior distributions are identical which generally means that data does not provide additional information. In the following we compare in more details the estimation results with the specified priors.

Habit persistence parameter is estimated to be much lower compared to our prior belief. This indicates that habit formation is not crucial for our model in order to replicate high frequency fluctuations of the Moroccan economy. The posterior mean for the elasticity of substitution between consumption and labor is close to the prior mean with a relatively tight probability interval. The use of consumption data series is necessary in order to get a better estimation of those parameters.

Our prior conviction about a relatively higher elasticity of substitution between tradable domestic and imported goods compared with the elasticity of substitution between tradables and non-tradables is confirmed by the data.

Estimation results also confirmed our prior belief about labor supply elasticity with a relatively tight probability interval. Also, the estimated value for the inverse elasticity of substitution between formal and informal labor is close to our prior belief which implies that the two types of labor are hardly substitutable. However, the credible interval is large. The inclusion of labor related data series could help pin down these two parameters more precisely.

Productivity persistence parameters are estimated to be higher than expected with a relatively higher persistence in the non-tradables compared with the tradables. The size of the productivity shock is much lower than expected for both sectors. The low size of tradables productivity shock despite the high volatility in tradables output observed in

the data may be due to the fact that part of this volatility is captured by the foreign demand shock. Again, adding labor related data series to the estimation could help pin down these two parameters more precisely.

The results of the estimation of Calvo parameters are interesting. Both parameters are well identified compared with the rest. The probability of price re-setting is highest for non-tradable firms with an average contract length around 1.3 quarters. The estimated average contract length for the tradables sector is around 9 quarters which indicates a high price stickiness. This means that fluctuations in the tradables sector are more persistent in response to shocks compared with the non-tradables sector.

Looking at posterior distributions of the reaction function parameters, the monetary authority seems to give priority to inflation targeting at the expense of output or real exchange rate targeting. The high coefficient placed on inflation fluctuations compared with the standard values in the literature may be the result of a shock absorbing role of the informal sector. In the presence of a significant informal sector, the central bank needs to be more aggressive in order to stabilize prices compared with its counterparts in developing economies with virtually inexistent informal sector. Controlling for fluctuations in output, inflation and the real exchange rate, the estimated persistence of the nominal interest rate is lower than expected. The estimated size of the interest rate shock is much lower than our prior in line with the low volatility of the interest rates observed in the data.

5.2 Main results

To assess the ability of our model to reproduce the stylized facts of Morocco's business cycle, we must take it through a series of empirical tests. The data generated by the model for its main variables should exhibit a satisfactory empirical fit to the observed data. Several methods can assess the empirical goodness of fit. We follow Cuche-Curti et al. (2009) and focus on 3 methods: unconditional second moments comparison (standard deviation, correlation and auto-correlation), historical simulations and the analysis of the Bayesian impulse-response functions (IRF).

5.2.1 Moments comparison

In order to assess the empirical fit of our model, we use the business cycle statistics reviewed in section 3.2. Table 5.4 shows a comparison between the empirical business cycle statistics and statistics computed from the estimated model using the smoothed variables produced by Dynare (Adjemian et al., 2011).

TABLE 5.4: Unconditional second moments comparison

	σ_X		σ_X/σ_Y		$\rho(X_t, X_{t-1})$		$\rho(X, Y)$	
	Data	Model	Data	Model	Data	Model	Data	Model
Y	1.51	1.39	1.00	1.00	0.20	0.26	1.00	1.00
Y_T	3.70	3.58	2.46	2.57	0.52	0.50	0.84	0.90
Y_N	1.51	1.00	1.00	0.72	0.29	0.31	0.21	0.06
C	2.02	1.37	1.34	0.99	0.15	0.12	0.20	0.03
π	0.28	0.28	0.18	0.20	0.49	0.48	0.02	-0.06
R	0.09	0.09	0.06	0.06	0.59	0.60	-0.27	-0.28
Q	1.52	1.52	1.01	1.09	0.71	0.72	-0.18	-0.07
E	1.10	3.05	0.73	2.19	0.76	0.79	-0.24	0.07
S	5.08	11.19	3.38	8.04	0.08	0.92	0.16	0.07

Regarding volatility, the model captures well both the absolute and relative levels for all variables except the terms of trade. The slightly lower volatility of the model's non-tradables output can be explained by the shock absorbing role of the informal sector and the same can be said for total consumption.

The persistence in the data is well captured by the model except for the terms of trade. The signs of the co-movements with output are well replicated except for inflation and the nominal exchange rate. The model is unable to replicate in a satisfying way the strength of correlation of some variables such as consumption with output.

Overall, the model does quite a good job in capturing the unconditional moments especially for the different output measures.

5.2.2 Historical simulations

Here we try to assess the ability of the model to simulate the fluctuations observed in the data for the macroeconomic variables of interest. We use the smoothed variables produced by Dynare (Adjemian et al., 2011) based on the estimated parameters presented above. Figure C.4 in appendix B presents the actual (blue line) and the simulated (dashed red line) fluctuations of output, consumption, inflation, the nominal interest rate and the real exchange rate.

The model replicates closely fluctuations in the tradables output. Differences between the simulated data and the actual data for total output and non-tradable output are mainly due to the inclusion of the informal sector in the model. Indeed, the two variables in the data correspond to their formal counterparts in the model. We can see that the simulated total output reacts less to negative shocks due to the shock absorbing capacity of the informal sector. The same goes for the simulated non-tradables output compared with the actual data series. Similarly, total consumption in the data corresponds only

to the formal sector while the model simulated series include both the formal and the informal sectors.

As we can see in figure C.4, the model was able to replicate some of the patterns observed in the consumption data despite the fact that it was not included in the estimation process.

As seen in figure C.4, the model also did a very good job in simulating the path of CPI inflation, the nominal interest rate and the real exchange rate. However the model did not perform well in replicating the variations in the data for the terms of trade.

5.2.3 Bayesian impulse-response functions

In this section we analyze the impact of informality on the model's behavior in response to shocks to productivity, interest rates, imported inflation and world consumption. The analyzed Bayesian impulse-response functions result from a temporary one standard deviation shock.

Figures C.5 to C.14 in appendix C show the results of the 5 shocks on all variables with (red line) and without (blue line) informal sector. To neutralize the presence of the informal sector we set its share in the goods and labor markets to 0 (i.e. $\omega = 0$ and $\kappa = 1$). The confidence intervals for the impulse-response functions with informal sector are reported in the figures and cover 95% of the probability mass.

In the following, we will start by analyzing the impulse-response functions resulting from the base model with informality and then comment when relevant the impact of the presence of the informal sector on the model's behavior.

5.2.3.1 Formal non-tradables productivity shock

The impulse-response functions resulting from a positive shock to the formal non-tradables firms productivity are presented in figures C.5 and C.6. Following the shock, formal non-tradable output increases along with the increase in productivity and the decrease in firms marginal costs. However, the increase in formal non-tradable output is lower than the increase in productivity since households are unwilling to absorb the surplus in production due to a combination of income and substitution effects. In the short run, the excess increase in productivity leads to a significant fall in formal labor and nominal wages along with a decrease in domestic goods prices. Formal real wages increase along with the increase in productivity. Because the same wage is payed by formal non-tradable firms to their workers, their marginal cost increases and causes a

spike in tradables inflation which repels consumers who choose instead to spend part of their additional income on imported goods. As a consequence, the domestic tradables sector suffers from a slow-down in consumption and output.

As an answer to the increase in the formal real wage, firms operating in the informal sector increase the informal real wage to keep their workers from going to the formal labor market. The increase in informal wages causes an increase in marginal costs. This crowding-out effect results in a negative supply shock which rises prices and decreases output and consumption. In addition, the informal sector suffers from households' substitution from informal to formal non-tradable goods as a result of the increase in informal prices.

The fall in domestic prices deteriorates the terms of trade long after the shock. The higher decrease in prices for non-tradable goods compared with tradable goods translates into an appreciation of the real exchange rate. In other words, the fall in domestic prices renders the domestic currency undervalued. Since the nominal exchange rate is fixed, the real exchange rate needs to appreciate in order to maintain the currency at its PPP value.

The monetary authority reacts to deflation by lowering interest rates. As soon as the effect of the productivity shock starts disappearing, the monetary authority rises slowly the short term interest rate to contain the inflationary pressures which sets back the economy into equilibrium.

By comparing the impulse-response functions with and without informality, one can clearly see that the presence of a large informal sector absorbs shocks in goods and labor markets and helps stabilizing the economy. Indeed, reducing the size of the informal sector causes an increase of the relative size of the formal non-tradables sector and shocks affecting it get amplified as a result. In other words, the informal sector reduces the impact of formal sector shocks on the whole economy.

5.2.3.2 Domestic tradables productivity shock

A shock to the productivity of domestic tradables firms triggers an increase in tradable output and consumption. However, domestic tradables consumption increases more than output. This can be explained by a fall in exports due to the observed appreciation of the real exchange rate.

In opposition with the crowding-out effect caused by the non-tradables productivity shock, a tradables productivity shock causes a crowding-in effect. Indeed, the low subtitutability between tradable and non-tradable goods not only prevents households from

switching their demand to tradable goods but also boosts the consumption of non-tradables thanks to the additional income generated by productivity gains.

The fact that the formal labor market is shared between tradable and formal non-tradable firms provides an interesting propagation mechanism between the two sectors. Indeed, the fall in real formal wages spreads to the formal non-tradables sector and reduces the marginal cost of its firms. This induces a fall in formal non-tradables prices which in addition to higher price stickiness in the tradables sector helps formal non-tradables firms benefit from a migration of consumers from other markets.

The formal non-tradables sector benefits also from the high substitution between formal and informal non-tradable goods. Indeed, additional demand goes to the formal sector as households switch from informal to formal goods to benefit from their lower prices.

Following the productivity shock, the demand for imports increases at first since domestic firms are unable to completely absorb the demand surge. But after 5 quarters, output catches up and households start switching back to local goods to benefit from their cheaper prices. Households also substitute from formal non-tradables to domestic tradables.

Facing a situation where output increases but prices fall, the central bank chooses to fight deflation by reducing short-term interest rates.

As opposed to the non-tradables productivity shock, here the presence of informality does not change significantly the reaction of the economy probably due to the absence of a direct transmission channel between the tradables and informal sectors.

5.2.3.3 Monetary policy shock

Figures C.9 and C.10 show the impulse-response functions resulting from a positive interest rate shock. This shock can be interpreted as a contractionary intervention of the central bank. As intended, the shock impacts negatively consumption, output, real wages and inflation. The non-tradables output recovers quickly after the shock due to its low level of price stickiness compared to the tradables sector which takes around a year to recover from the shock. In other words, the higher fall in prices in the formal non-tradables sector attracts households' demand away from the tradables sector which explains the surge in formal non-tradables consumption and output.

Following the shock, the real exchange rate appreciates and the terms of trade deteriorate because of the fall in local prices compared with foreign prices. The appreciation in the real exchange rate causes a fall in exports and a slight increase in domestic tradables consumption.

Inflation returns after 1 or 2 quarters to its steady state then overshoots slightly to converge to equilibrium after 1 year.

Here again, the presence of an informal sector does not alter significantly the behavior of the economy.

5.2.3.4 Imported inflation shock

A one standard deviation positive shock to world prices increases inflation significantly which amounts to a negative wealth shock. As a consequence, consumption and output decrease in all sectors of the economy.

The tradable sector recovers quickly from the shock thanks to a substitution from expensive imports to cheaper domestic tradable goods. This is visible from the gradual deterioration in the terms of trade. However, exports fall significantly due to the appreciation of the real exchange rate.

Meanwhile, higher import prices coupled with the resulting increase in demand for domestic tradable goods push up CPI inflation. These inflationary pressures trigger the reaction of the monetary authority. The nominal interest rate increases quickly as a response to the increase in inflation. The increase in interest rates further reduces consumption and output in the non-tradables sectors. As a reaction to the activity slowdown reinforced by its monetary policy, the central bank starts lowering its interest rate to put back the economy in its steady state.

One can observe that despite a fall in the non-tradables sectors, the central bank proceeds to rising the short-term interest rate to fight against imported inflation which strongly delays the recovery of local demand after the initial shock. This may be an argument against CPI inflation targeting which could be replaced by domestic inflation as the main target for price stabilization policy to avoid unjustified reactions to imported inflation. This issue can be explored in more details by comparing the welfare effect of monetary policy reaction function based on the two inflation measures.

With this kind of negative demand shocks, the presence of informality appears to have no effect on the behavior of the economy.

5.2.3.5 Foreign demand shock

Following a positive shock to foreign consumption, tradables output increases substantially due to a increased exports. This creates pressure on formal labor market which causes real formal wages to increase. Also, formal labor supply shifts from non-tradables

to tradables sectors because of the mobility between the two labor markets. This shift causes the non-tradables output to decrease.

In addition, the rise in real formal wages causes prices of non-tradable goods to increase which pushes households to shift their demand to cheaper informal goods. As a consequence, consumption and output increase in the informal non-tradables sector.

The central bank reacts to the strong increase in tradables output with a mild increase in interest rates.

The terms of trade deteriorate and the real exchange rate appreciates due to a higher decrease in non-tradable goods prices compared with tradable goods prices.

The presence of the informal sector absorbs part of the shock and reduces the size of the monetary intervention needed to stabilize the economy.

Chapter 6

Conclusion

In this thesis, we try to understand the role of some macroeconomic features specific to developing economies, in particular the presence of a large informal sector, in shaping the reaction of the economy to different shocks. For this, we build a multi-sector small open economy DSGE model incorporating some of the distinctive features of developing countries. The model distinguishes between tradable and non-tradable sectors with the latter divided into formal and informal sub-sectors. The informal sector is modeled as simple as possible with flexible goods and labor markets in line with the main empirical findings. In the formal sectors, the model features imperfect competition in goods and labor markets and nominal rigidities in prices à la Calvo (1983). In addition, the model incorporates a fixed exchange regime and a monetary authority reacting to fluctuations in inflation, output and the real exchange rate.

To calibrate and estimate the model, we use quarterly data from Morocco. The model's parameters were separated into three groups. The first group of parameters is calibrated using long-term averages taken from the data and available surveys of the informal sector. The second group consists of auto-regressive parameters and is fitted from the data using OLS. The third group is mainly composed of deep behavioral parameters and is estimated using a Bayesian estimation approach. A substantial effort was spent on the documentation of the choice of prior distributions. We use the longest available quarterly data series on tradable and non-tradable outputs, CPI inflation, the nominal interest rate and the real exchange rate. The data series were log-scaled and detrended using the HP-filter.

The model generally fits the data well and is able to replicate the stylized facts of Morocco's business cycle. The estimation results are in line with the literature except for the monetary policy coefficients. In particular, the coefficient on inflation in the reaction function is much higher compared with standard values. Also, the central

bank puts more weight on fluctuations in inflation compared with fluctuations in output and the real exchange rate. The estimation results show also a high price stickiness in the tradables sector with an average contract duration of 9 quarters. The elasticity of substitution is estimated to be higher between non-tradable formal and informal goods than between tradable and non-tradable goods. Habit formation is estimated to have a negligible role in the economy.

The study of the impulse-response functions shows different crowding-in and crowding-out effects depending on whether the productivity shock affects the non-tradables or the tradables sectors. The presence of an informal sector plays the role of a buffer which reduces the impact of formal sector shocks on the whole economy. This is observed in particular for productivity shocks. However, no evidence is found of a shock absorbing role of the informal sector in the case of interest rate or foreign demand shocks. The shared formal labor market provides a direct transmission mechanism between the tradables and formal non-tradables sector. In addition, the fixed exchange regime gives place to interesting interactions between the real exchange rate and the terms of trade based on the price differential between tradable and non-tradable goods.

To our knowledge, this is the first work that uses a Bayesian framework for the estimation of a DSGE model based on Morocco quarterly data. In future research, we plan to extend the model by adding capital and investment, incomplete pass-through, nominal rigidities in wages, introducing a banking sector and government and explicitly modeling the World economy. Other features specific to developing economies will be added such as remittances, natural resources sector and constraints on credit and foreign exchange reserves. This will enable us to better replicate stylized facts of developing economies and study the role of each of these features in shaping the reaction of the economy to different shocks in a more detailed way.

Appendix A

Log-linearized model

A.1 Shocks

- Productivity shocks: $\varepsilon_{a_{T,t}}, \varepsilon_{a_{N,t}}$
- Interest rate shock: ε_r
- Foreign demand shock: ε_{c^*}
- Imported inflation shock: ε_{π^*}

A.2 Equations

- **Aggregate output**

$$\hat{y}_t = (1-\lambda)\hat{y}_{T,t} + \lambda\hat{y}_{N,t} \tag{A.2.1}$$

- **Tradable output**

$$\hat{y}_{T,t} = \hat{a}_{T,t} + \hat{n}_{T,t} \tag{A.2.2}$$

- **Non-tradable output**

$$\hat{y}_{N,t} = (1-\omega)\hat{y}^F_{N,t} + \omega\hat{y}^I_{N,t} \tag{A.2.3}$$

- **Formal non-tradable output**

$$\hat{y}^F_{N,t} = \hat{a}_{N,t} + \hat{n}^F_{N,t} \tag{A.2.4}$$

- **Informal non-tradable output**

$$\hat{y}^I_{N,t} = \hat{n}^I_{N,t} \tag{A.2.5}$$

- **Domestic tradable firms productivity**

$$\hat{a}_{T,t} = \rho_{a_{T,t}} \hat{a}_{T,t-1} + \varepsilon_{a_{T,t}} \tag{A.2.6}$$

where $\varepsilon_{a_{T,t}} \sim \mathcal{N}(O, \sigma_{a_T})$.

- **Non-tradable formal firms productivity**

$$\hat{a}_{N,t} = \rho_{a_{N,t}} \hat{a}_{N,t-1} + \varepsilon_{a_{N,t}} \tag{A.2.7}$$

where $\varepsilon_{a_{N,t}} \sim \mathcal{N}(O, \sigma_{a_N})$.

- **Domestic tradable firms marginal cost**

$$\hat{mc}_{T,t} = \hat{w}^F_t - \hat{p}^H_{T,t} - \hat{a}_{T,t} \tag{A.2.8}$$

- **Non-tradable formal firms marginal cost**

$$\hat{mc}_{N,t} = \hat{w}^F_t - \hat{p}^F_{N,t} - \hat{a}_{N,t} \tag{A.2.9}$$

- **Aggregate Consumption**

$$\hat{c}_t = h\hat{c}_{t-1} - \frac{1-h}{\sigma}(\varphi \hat{n}_t - \hat{w}_t + \hat{p}_t) \tag{A.2.10}$$

- **Euler equation**

$$\hat{c}_t = \frac{h}{1+h}\hat{c}_{t-1} + \frac{1}{1+h}\mathrm{E}_t[\hat{c}_{t+1}] - \frac{1-h}{\sigma(1+h)}(\hat{r}_t - \mathrm{E}_t[\pi_{t+1}]) \tag{A.2.11}$$

- **Tradables consumption**

$$\hat{c}_{T,t} = -\nu\left(\hat{p}_{T,t} - \hat{p}_t\right) + \hat{c}_t \tag{A.2.12}$$

- **Foreign tradables consumption**

$$\hat{c}^F_{T,t} = -\eta\left(\hat{p}^F_{T,t} - \hat{p}_{T,t}\right) + \hat{c}_{T,t} \tag{A.2.13}$$

- **Domestic tradables consumption**

$$\hat{c}^H_{T,t} = -\eta \left(\hat{p}^H_{T,t} - \hat{p}_{T,t}\right) + \hat{c}_{T,t} \tag{A.2.14}$$

- **Non-tradables consumption**

$$\hat{c}_{N,t} = -\nu \left(\hat{p}_{N,t} - \hat{p}_t\right) + \hat{c}_t \tag{A.2.15}$$

- **Formal non-tradables consumption**

$$\hat{c}^F_{N,t} = -\mu \left(\hat{p}^F_{N,t} - \hat{p}_{N,t}\right) + \hat{c}_{N,t} \tag{A.2.16}$$

- **Informal non-tradables consumption**

$$\hat{c}^I_{N,t} = -\mu \left(\hat{p}^I_{N,t} - \hat{p}_{N,t}\right) + \hat{c}_{N,t} \tag{A.2.17}$$

- **CPI Inflation**

$$\pi_t = (1-\lambda)\pi_{T,t} + \lambda\pi_{N,t} \tag{A.2.18}$$

- **Domestic Inflation**

$$\pi^d_t = (1-\lambda)\pi^H_{T,t} + \lambda\pi_{N,t} \tag{A.2.19}$$

- **Tradables inflation**

$$\pi_{T,t} = (1-\alpha)\pi^H_{T,t} + \alpha\pi^F_{T,t} \tag{A.2.20}$$

- **Non-tradables inflation**

$$\pi_{N,t} = (1-\omega)\pi^F_{N,t} + \omega\pi^I_{N,t} \tag{A.2.21}$$

- **Domestic tradables NKPC**

$$\pi^H_{T,t} = \beta \mathrm{E}_t[\pi^H_{T,t+1}] + \lambda_T \hat{mc}_{T,t} \tag{A.2.22}$$

  where $\lambda_T \equiv \frac{(1-\theta_T)(1-\beta\theta_T)}{\theta_T}$

- **Imported inflation**

$$\hat{\pi}^F_{T,t} = \Delta\hat{e}_t + \hat{\pi}^*_t \tag{A.2.23}$$

- **Formal non-tradables NKPC**

$$\pi^F_{N,t} = \beta \mathrm{E}_t[\pi^F_{N,t+1}] + \lambda_N \hat{mc}_{N,t} \tag{A.2.24}$$

where $\lambda_N \equiv \frac{(1-\theta_N)(1-\beta\theta_N)}{\theta_N}$

- **Informal non-tradables inflation**

$$\hat{\pi}^I_{N,t} = \hat{w}^I_t - \hat{w}^I_{t-1} \tag{A.2.25}$$

- **CPI price level**

$$\hat{p}_t = \hat{p}_{t-1} + \pi_t \tag{A.2.26}$$

- **Tradables price level**

$$\hat{p}_{T,t} = \hat{p}_{T,t-1} + \pi_{T,t} \tag{A.2.27}$$

- **Domestic tradables price level**

$$\hat{p}^H_{T,t} = \hat{p}^H_{T,t-1} + \pi^H_{T,t} \tag{A.2.28}$$

- **Foreign tradables price level**

$$\hat{p}^F_{T,t} = \hat{p}^F_{T,t-1} + \pi^F_{T,t} \tag{A.2.29}$$

- **Non-tradables price level**

$$\hat{p}_{N,t} = \hat{p}_{N,t-1} + \pi_{N,t} \tag{A.2.30}$$

- **Formal non-tradables price level**

$$\hat{p}^F_{N,t} = \hat{p}^F_{N,t-1} + \pi^F_{N,t} \tag{A.2.31}$$

- **Informal non-tradables price level**

$$\hat{p}^I_{N,t} = \hat{p}^I_{N,t-1} + \pi^I_{N,t} \tag{A.2.32}$$

- **Aggregate supply of labor**

$$\hat{n}_t = \kappa^{-\tau} \left(\frac{N^F}{N} \right)^{1+\tau} \hat{n}_t^F + (1-\kappa)^{-\tau} \left(\frac{N^I}{N} \right)^{1+\tau} \hat{n}_t^I \tag{A.2.33}$$

- **Supply of formal labor**

$$\hat{n}_t^F = \frac{1}{\tau} \left(\hat{w}_t^F - \hat{w}_t \right) + \hat{n}_t \tag{A.2.34}$$

- **Supply of informal labor**

$$\hat{n}_t^I = \frac{1}{\tau} \left(\hat{w}_t^I - \hat{w}_t \right) + \hat{n}_t \tag{A.2.35}$$

- **Nominal exchange rate equation**

$$\hat{e}_t = \rho_e \hat{e}_{t-1} + \varepsilon_{e,t} \tag{A.2.36}$$

where $\varepsilon_{uip,t} \sim \mathcal{N}(O, \sigma_{uip})$.

- **Terms of trade**

$$\hat{s}_t = \hat{e}_t + \hat{p}_t^* - \hat{p}_{T,t}^H \tag{A.2.37}$$

- **Real exchange rate**

$$\hat{q}_t = (1-\alpha)\hat{s}_t + \lambda(\hat{p}_{N,t} - \hat{p}_{T,t}) \tag{A.2.38}$$

- **Monetary policy**

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1-\rho_r)(\psi_y \hat{y}_t + \psi_\pi \pi_t + \psi_e \hat{e}_t) + \varepsilon_{r,t} \tag{A.2.39}$$

- **Goods markets clearing conditions**

$$\hat{y}_{T,t} = (1-\iota)\hat{c}_{T,t}^H + \iota \left(-\gamma(\hat{p}_{T,t}^H - \hat{p}_t) + \gamma \hat{q}_t + \hat{c}_t^* \right) \tag{A.2.40}$$

$$\hat{y}_{N,t}^F = \hat{c}_{N,t}^F \tag{A.2.41}$$

$$\hat{y}_{N,t}^I = \hat{c}_{N,t}^I \tag{A.2.42}$$

- **Formal labor market clearing condition**

$$\hat{n}_t^F = \chi \hat{n}_{T,t} + (1-\chi)\,\hat{n}_{N,t}^F \tag{A.2.43}$$

- **World economy**

  – World output

$$\hat{c}_t^* = \rho_{c^*}\hat{c}_{t-1}^* + \epsilon_{c^*,t} \tag{A.2.44}$$

  – World inflation

$$\pi_t^* = \rho_{\pi^*}\pi_{t-1}^* + \epsilon_{\pi^*,t} \tag{A.2.45}$$

$$\hat{p}_t^* = \hat{p}_{t-1}^* + \pi_t^* \tag{A.2.46}$$

  – World interest rate

$$\hat{r}_t^* = \rho_{r^*}\hat{r}_{t-1}^* + \epsilon_{r^*,t} \tag{A.2.47}$$

Appendix B

Mathematical derivations

B.1 Households utility maximization problem

We consider the following form for the utility function:

$$U(C,N) \equiv \frac{\tilde{C}^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi} \tag{B.1.1}$$

where σ is the inverse of the elasticity of substitution between consumption and labor and φ is the inverse labor elasticity.

The household maximizes his lifetime utility:

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\tilde{C}_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \tag{B.1.2}$$

subject to the inter-temporal budget constraint:

$$P_t C_t + \mathrm{E}_t[Q_{t,t+1} D_{t+1}] \leq D_t + W_t N_t + \Pi_t \tag{B.1.3}$$

We have the following Lagrangian:

$$\mathcal{L} = \mathrm{E} \sum_{t=0}^{\infty} \left[\beta^t \left(\frac{\tilde{C}_t^{\,1-\sigma}}{1-\sigma} - \frac{N_t^{\,1+\varphi}}{1+\varphi} \right) - \lambda_t \left[P_t C_t + \mathrm{E}_t[Q_{t,t+1} D_{t+1}] - D_t - W_t N_t - \Pi_t \right] \right]$$

The first order conditions are as follows:

$$C_t : \beta^t \tilde{C}_t^{-\sigma} = \lambda_t P_t \tag{B.1.4}$$

$$N_t : \beta^t N_t^{\varphi} = \lambda_t W_t \tag{B.1.5}$$

$$D_{t+1} : \lambda_{t+1} = \lambda_t \mathrm{E}_t[Q_{t,t+1}] \tag{B.1.6}$$

Combining (B.1.4) and (B.1.5), one gets the usual intratemporal optimality condition:

$$\frac{N_t^{\varphi}}{\tilde{C}_t^{-\sigma}} = \frac{W_t}{P_t} \tag{B.1.7}$$

This condition states that the optimal labor-leisure choice requires that the marginal rate of substitution between consumption and hours worked (also leisure) equals the real wage.

Combining (B.1.4) and (B.1.6), one gets:

$$\beta^{t+1} \mathrm{E}_{t+1} \left[\frac{\tilde{C}_{t+1}^{-\sigma}}{P_{t+1}} \right] = \beta^t \frac{\tilde{C}_t^{-\sigma}}{P_t} \mathrm{E}_t[Q_{t,t+1}]$$

This expression can be rewritten to obtain the following intertemporal optimality condition (Euler equation):

$$\beta R_t \mathrm{E}_t \left[\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right] = 1 \tag{B.1.8}$$

where $R_t = \frac{1}{\mathrm{E}_t[Q_{t,t+1}]}$ is the one-period gross return on the risk-free bond denominated in the domestic currency.

Log-linearizing the optimality conditions and the expression for habit formation, one gets:

$$\sigma \tilde{c}_t + \varphi n_t = w_t - p_t \tag{B.1.9}$$

$$\tilde{c}_t = \mathrm{E}_t[\tilde{c}_{t+1}] - \frac{1}{\sigma}(r_t - \mathrm{E}_t[\pi_{t+1}]) \tag{B.1.10}$$

$$\tilde{c}_t = \frac{c_t - hc_{t-1}}{1-h} \tag{B.1.11}$$

Combining (B.1.11) with both (B.1.9) and (B.1.10), one can write the following equations:

$$\frac{\sigma}{1-h}(c_t - hc_{t-1}) + \varphi n_t = w_t - p_t \tag{B.1.12}$$

$$c_t = \frac{h}{1+h}c_{t-1} + \frac{1}{1+h}\mathrm{E}_t[c_{t+1}] - \frac{1-h}{\sigma(1+h)}(r_t - \mathrm{E}_t[\pi_{t+1}]) \tag{B.1.13}$$

B.2 Optimal price setting

Let $\bar{P}^H_{T,t}(j)$ be the price set by a firm adjusting its price in period t. Under the Calvo price setting, $P^H_{T,t+k}(j) = \bar{P}^H_{T,t}(j)$ with probability θ^k_T for $k = 0, 1, 2, \ldots$. For convenience, we drop the j subscript since all re-optimizing firms will choose the same price.

The profit maximization problem can be expressed as follows:

$$max_{\bar{P}^H_{T,t}} \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k_T \mathrm{E}_t \left[Q_{t,t+k} \left(Y_{T,t+k}(\bar{P}^H_{T,t} - MC^n_{T,t+k}) \right) \right] \tag{B.2.1}$$

subject to the assumption of Calvo pricing and the following demand function:

$$Y_{T,t+k}(j) = \left(\frac{\bar{P}^H_{T,t}(j)}{P^H_{T,t+k}} \right)^{-\varepsilon} \left(C^H_{T,t+k} + \int_0^1 C^{H,i}_{T,t+k} \, \mathrm{d}i \right) \tag{B.2.2}$$

where $MC^n_{T,t} = \frac{W_{T,t}}{A_{T,t}}$ is the nominal marginal cost.

(4.56) uses the stochastic discount factor $Q_{t,t+k}$ since firms have to take into account the future demand elasticities when setting prices.

Substituting the demand curve (4.57) in (4.56) and solving for the first order condition with respect to $\bar{P}^H_{T,t}$ leads to:

$$\sum_{k=0}^{\infty}\theta_T^k \mathrm{E}_t\left[Q_{t,t+k}\left(-\varepsilon\frac{Y_{T,t+k}}{\bar{P}_{T,t}^H}(\bar{P}_{T,t}^H - MC_{T,t+k}^n) + Y_{T,t+k}\right)\right] = 0$$

$$\sum_{k=0}^{\infty}\theta_T^k \mathrm{E}_t\left[Q_{t,t+k}\left((1-\varepsilon)Y_{T,t+k} + \varepsilon Y_{T,t+k}\frac{MC_{T,t+k}^n}{\bar{P}_{T,t}^H}\right)\right] = 0$$

$$\sum_{k=0}^{\infty}\theta_T^k \mathrm{E}_t\left[Q_{t,t+k}Y_{T,t+k}\left(\bar{P}_{T,t}^H - \frac{\varepsilon}{\varepsilon-1}MC_{T,t+k}^n\right)\right] = 0$$

We replace by the expression for the stochastic discount factor, $Q_{t,t+k} = \beta^k \mathrm{E}_t\left[\left(\frac{\tilde{C}_{t+k}}{\tilde{C}_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+k}}\right)\right]$, to get:

$$\sum_{k=0}^{\infty}\theta_T^k \mathrm{E}_t\left[\beta^k \mathrm{E}_t\left[\left(\frac{\tilde{C}_{t+k}}{\tilde{C}_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+k}}\right)\right]Y_{T,t+k}\left(\bar{P}_{T,t}^H - \frac{\varepsilon}{\varepsilon-1}MC_{T,t+k}^n\right)\right] = 0$$

$$\sum_{k=0}^{\infty}\beta^k\theta_T^k \mathrm{E}_t\left[\frac{\tilde{C}_{t+k}^{-\sigma}}{P_{t+k}}Y_{T,t+k}\left(\bar{P}_{T,t}^H - \frac{\varepsilon}{\varepsilon-1}MC_{T,t+k}^n\right)\right] = 0$$

$$\sum_{k=0}^{\infty}\beta^k\theta_T^k \mathrm{E}_t\left[\tilde{C}_{t+k}^{-\sigma}Y_{T,t+k}\frac{P_{T,t-1}^H}{P_{t+k}}\left(\frac{\bar{P}_{T,t}^H}{P_{T,t-1}^H} - \frac{\varepsilon}{\varepsilon-1}\frac{P_{T,t+k}^H}{P_{T,t-1}^H}\frac{MC_{T,t+k}^n}{P_{T,t+k}^H}\right)\right] = 0$$

Substituting by $\Pi_{T,t-1,t+k}^H \equiv \frac{P_{T,t+k}^H}{P_{T,t-1}^H}$ and $MC_{T,t+k} = \frac{MC_{T,t+k}^n}{P_{T,t+k}^H}$, one gets:

$$\sum_{k=0}^{\infty}\beta^k\theta_T^k \mathrm{E}_t\left[\tilde{C}_{t+k}^{-\sigma}Y_{T,t+k}\frac{P_{T,t-1}^H}{P_{t+k}}\left(\frac{\bar{P}_{T,t}^H}{P_{T,t-1}^H} - \frac{\varepsilon}{\varepsilon-1}\Pi_{T,t-1,t+k}^H MC_{T,t+k}\right)\right] = 0$$

Log-linearizing the previous first order condition around the zero inflation steady state ($\Pi_{T,t,t+k} = \Pi = 1$), we get:

$$\bar{p}_{T,t}^H = p_{T,t-1}^H + \sum_{k=0}^{\infty}\beta^k\theta_T^k \mathrm{E}_t\left[\pi_{T,t+k}^H\right] + (1-\beta\theta_T)\sum_{k=0}^{\infty}\beta^k\theta_T^k \mathrm{E}_t\left[mc_{T,t+k} - mc_T\right]$$

where $mc_T = \log MC_T = \log\frac{\varepsilon-1}{\varepsilon}$ is the steady state real marginal cost.

The previous equation involves an infinite sum and it describes the standard solution to the following first-order stochastic difference equation:

$$\bar{p}_{T,t}^H - p_{T,t-1}^H = \beta\theta_T \mathrm{E}_t\left[\bar{p}_{T,t+1}^H - p_{T,t}^H\right] + \pi_{T,t}^H + (1-\beta\theta_T)\left(mc_{T,t} - mc_T\right) \qquad \text{(B.2.3)}$$

Under the Calvo price setting defined above, the domestic tradable goods price index is expressed by:

$$P_{T,t}^{H} \equiv \left[\theta_T (P_{T,t}^{H})^{1-\varepsilon} + (1-\theta_T)(\bar{P}_{T,t}^{H})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{B.2.4}$$

Log-linearizing this expression around the zero inflation steady state, one gets:

$$\pi_{T,t}^{H} = (1-\theta_T)(\bar{p}_{T,t}^{H} - p_{T,t-1}^{H}) \tag{B.2.5}$$

Combining equations (B.2.3) and (B.2.5):

$$\begin{aligned}
\frac{\pi_{T,t}^{H}}{1-\theta_T} &= \beta\theta_T \mathrm{E}_t\left[\frac{\pi_{T,t+1}^{H}}{1-\theta_T}\right] + \pi_{T,t}^{H} + (1-\beta\theta_T)\left(mc_{T,t} - mc_T\right) \\
\pi_{T,t}^{H} &= \beta\theta_T \mathrm{E}_t\left[\pi_{T,t+1}^{H}\right] + (1-\theta_T)\pi_{T,t}^{H} + (1-\theta_T)(1-\beta\theta)\left(mc_{T,t} - mc_T\right) \\
\theta_T\pi_{T,t}^{H} &= \beta\theta_T \mathrm{E}_t\left[\pi_{T,t+1}^{H}\right] + (1-\theta_T)(1-\beta\theta_T)\left(mc_{T,t} - mc_T\right) \\
\pi_{T,t}^{H} &= \beta \mathrm{E}_t\left[\pi_{T,t+1}^{H}\right] + \frac{(1-\theta_T)(1-\beta\theta_T)}{\theta_T}\left(mc_{T,t} - mc_T\right)
\end{aligned}$$

Which gives us the following New Keynesian Philips Curve for the tradables sector:

$$\pi_{T,t}^{H} = \beta \mathrm{E}_t\left[\pi_{T,t+1}^{H}\right] + \lambda_T\left(mc_{T,t} - mc_T\right) \tag{B.2.6}$$

where $\lambda_T = \frac{(1-\theta_T)(1-\beta\theta_T)}{\theta_T}$.

Following the same steps for the formal non-tradables sector, one obtains a similar New Keynesian Philips Curve:

$$\pi_{N,t}^{F} = \beta \mathrm{E}_t[\pi_{N,t+1}^{F}] + \lambda_N\left[mc_{N,t} - mc_N\right] \tag{B.2.7}$$

where $\lambda_N \equiv \frac{(1-\theta_N)(1-\beta\theta_N)}{\theta_N}$

Appendix C

Figures for model diagnostic and results

FIGURE C.1: Prior and posterior distributions (1)

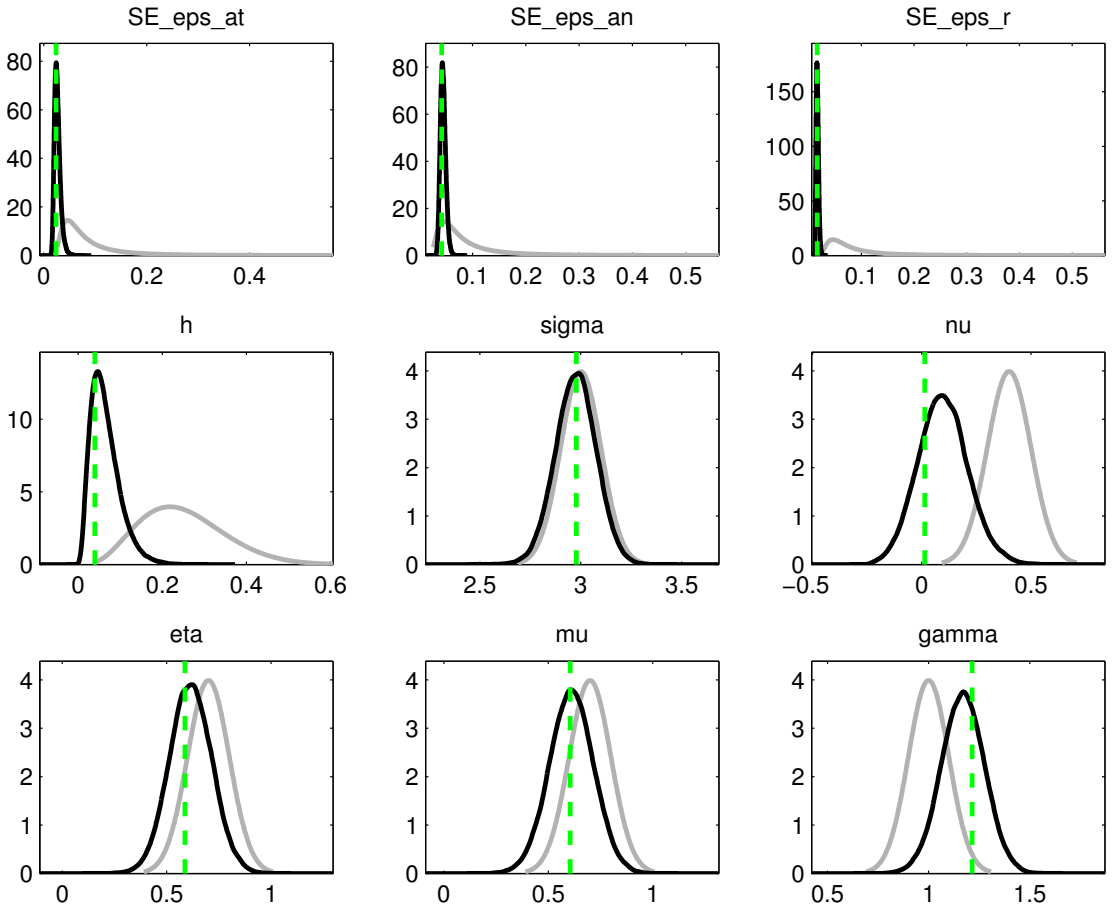


FIGURE C.2: Prior and posterior distributions (2)

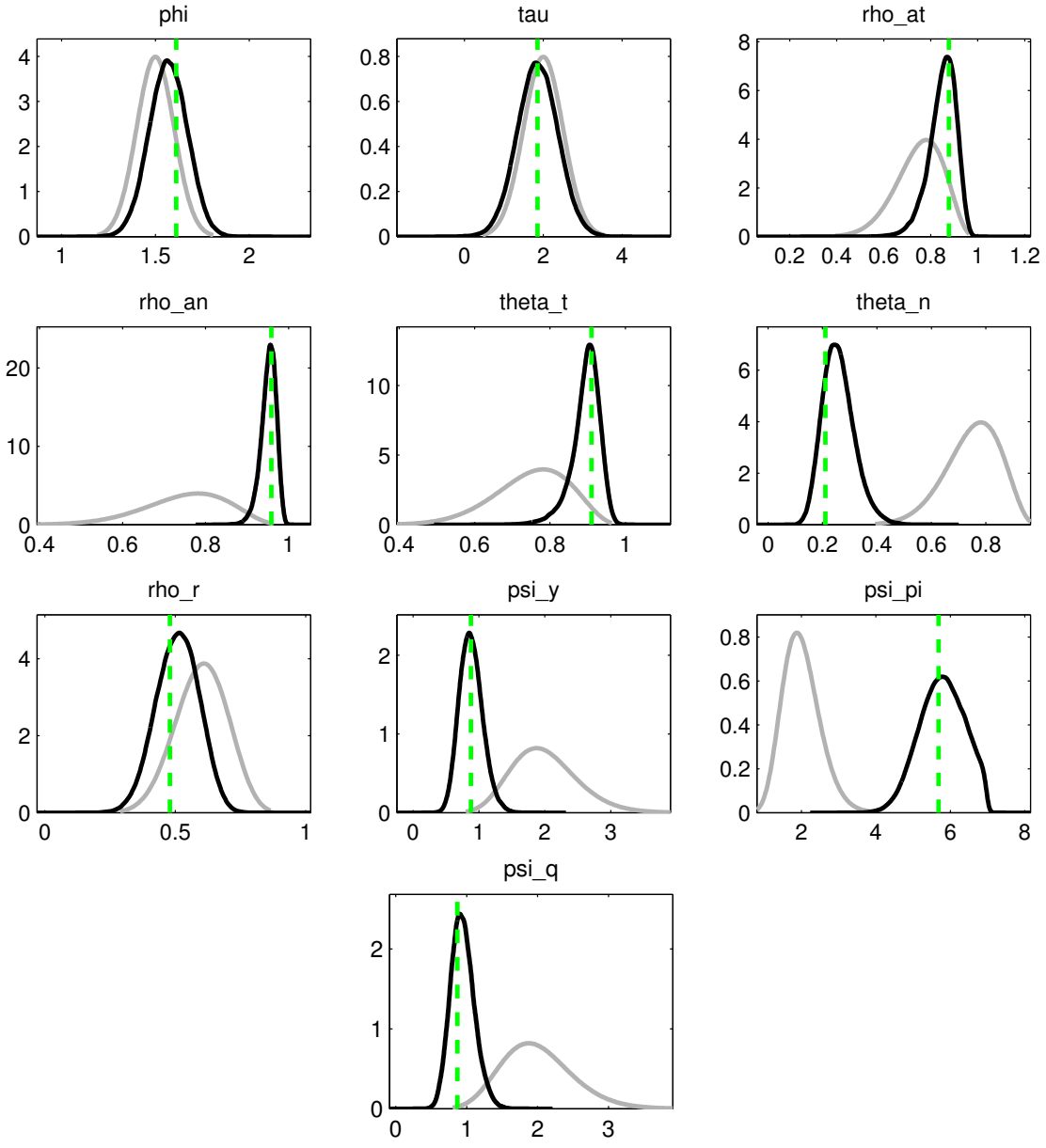


FIGURE C.3: Identification and sensitivity

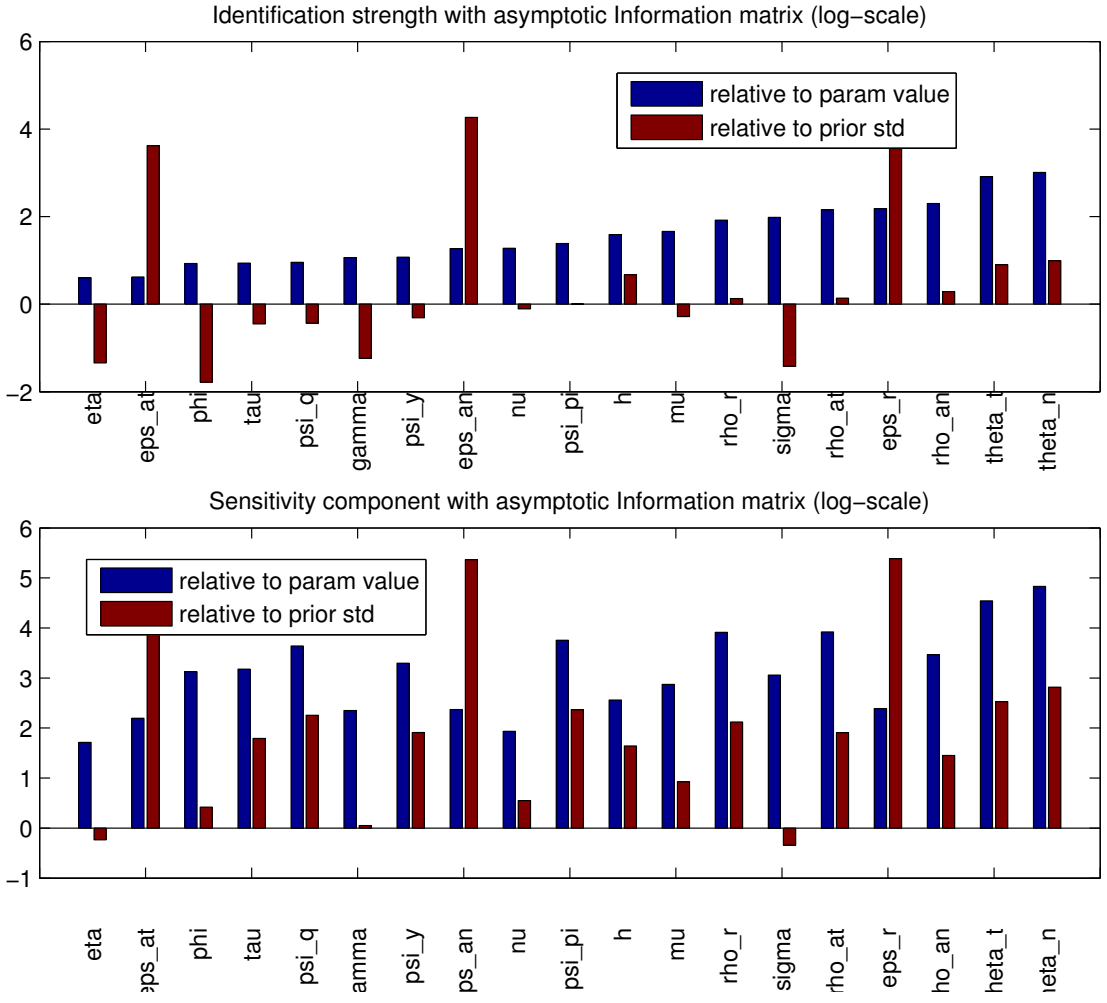


FIGURE C.4: Historical simulations

FIGURE C.5: Formal non-tradables productivity shock (1)

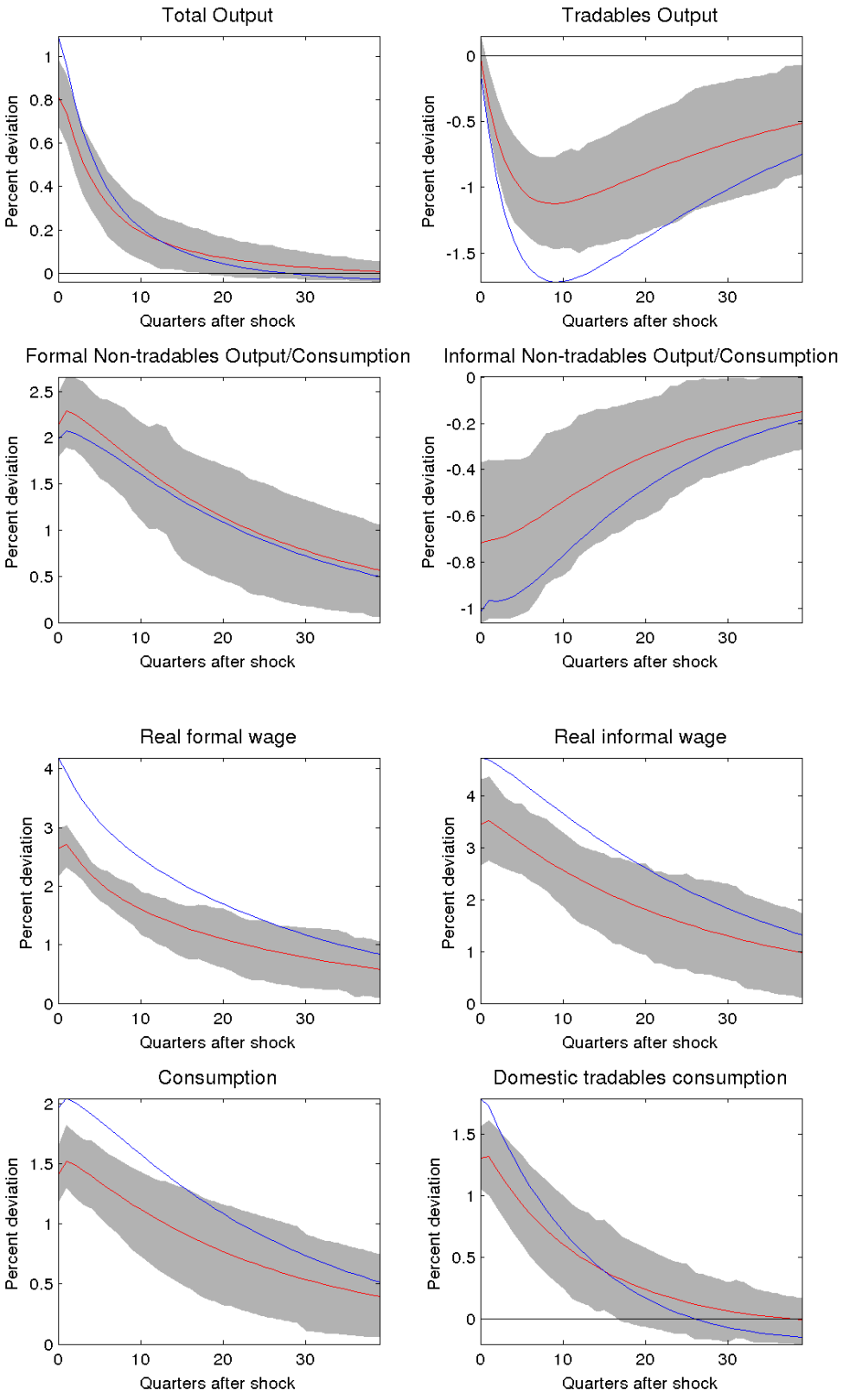


FIGURE C.6: Formal non-tradables productivity shock (2)

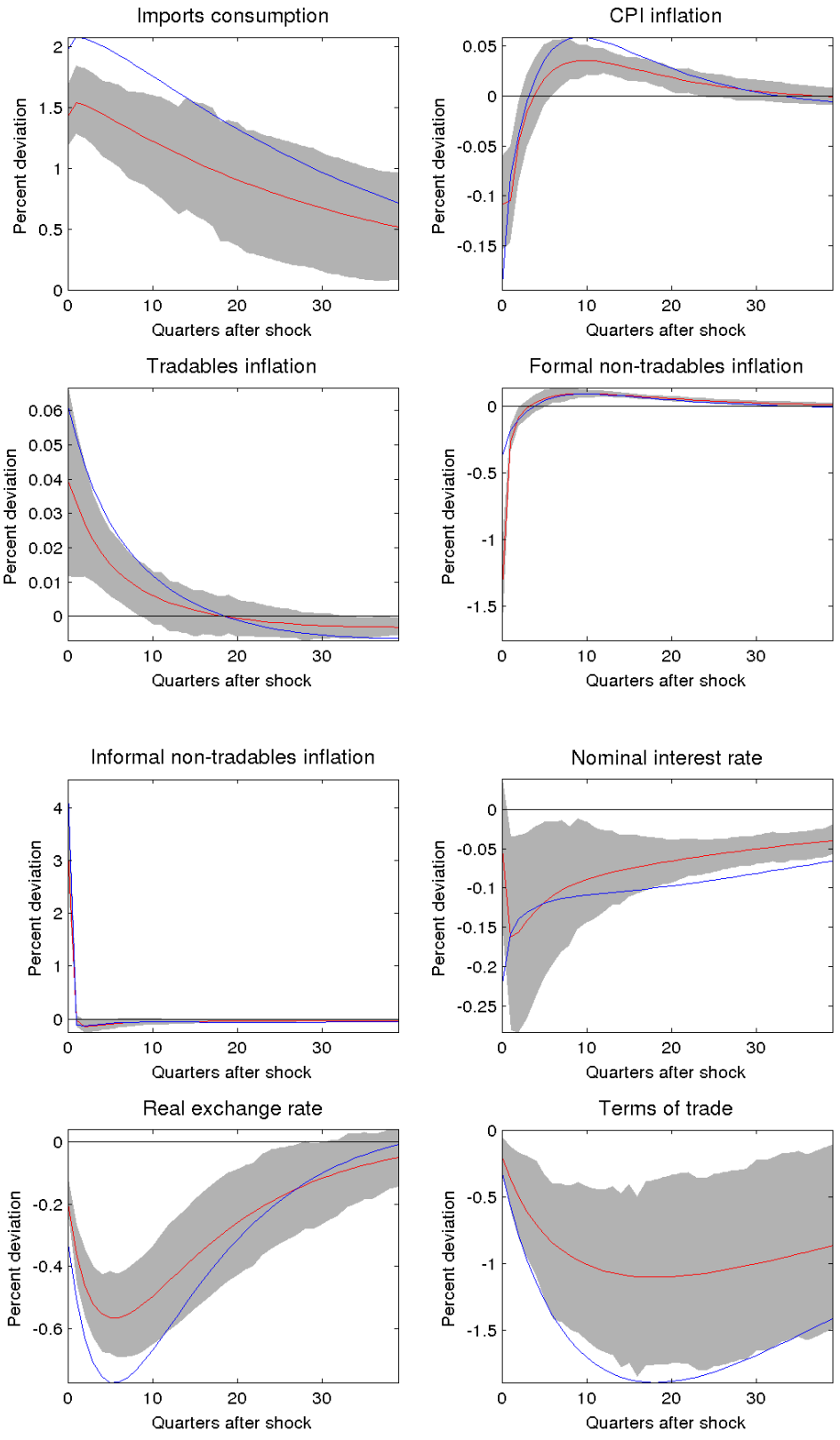


FIGURE C.7: Domestic tradables productivity shock (1)

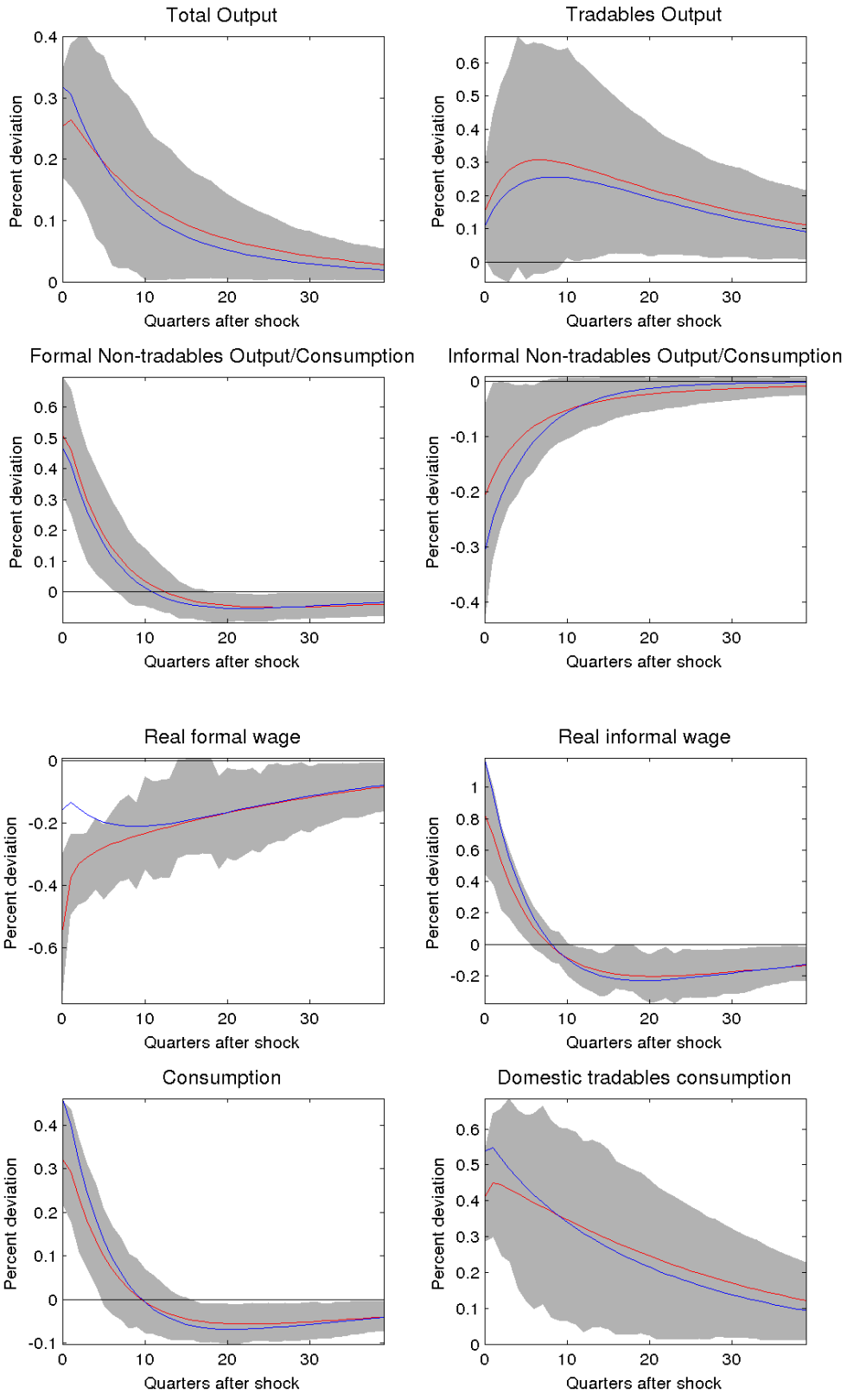


FIGURE C.8: Domestic tradables productivity shock (2)

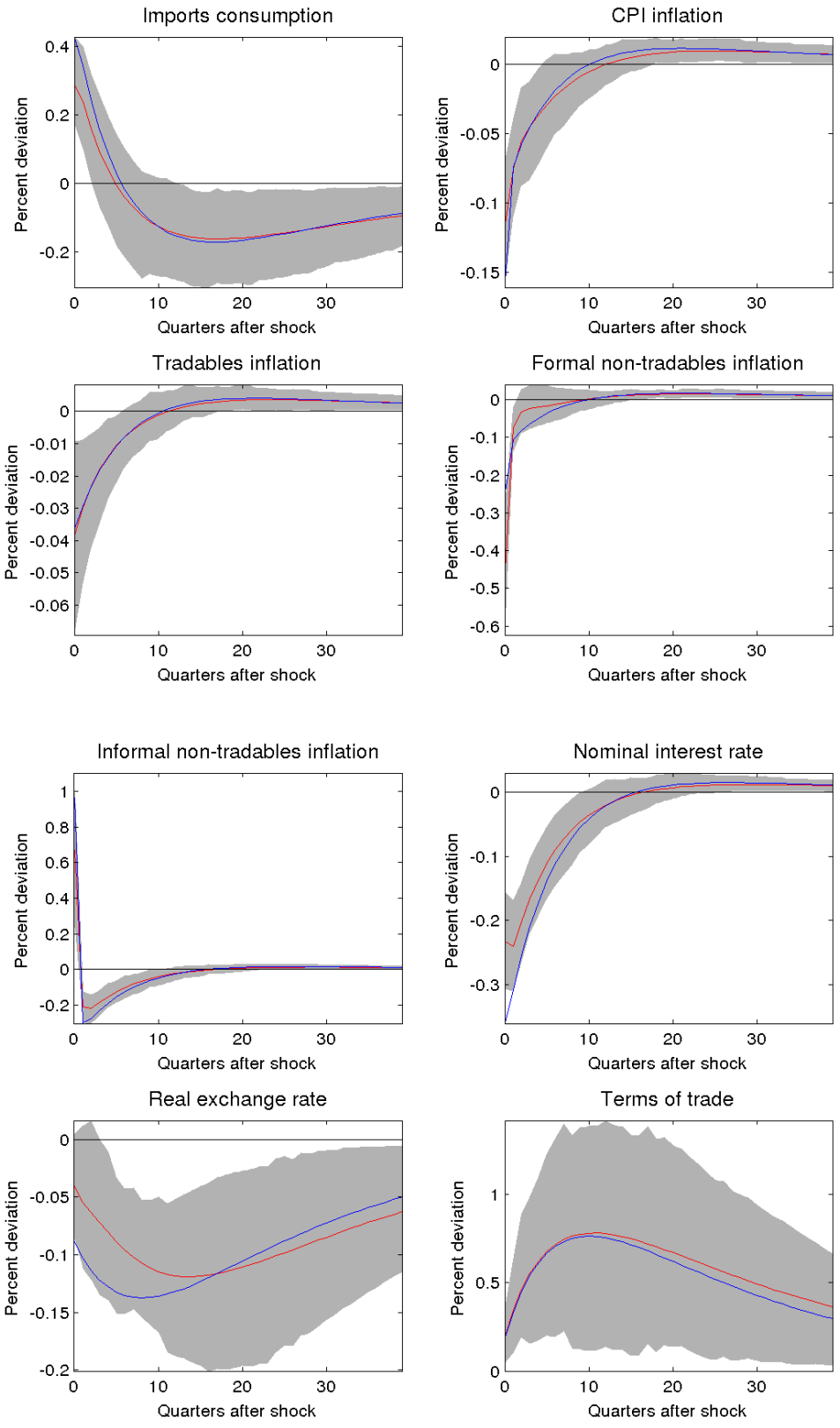


FIGURE C.9: Interest rate shock (1)

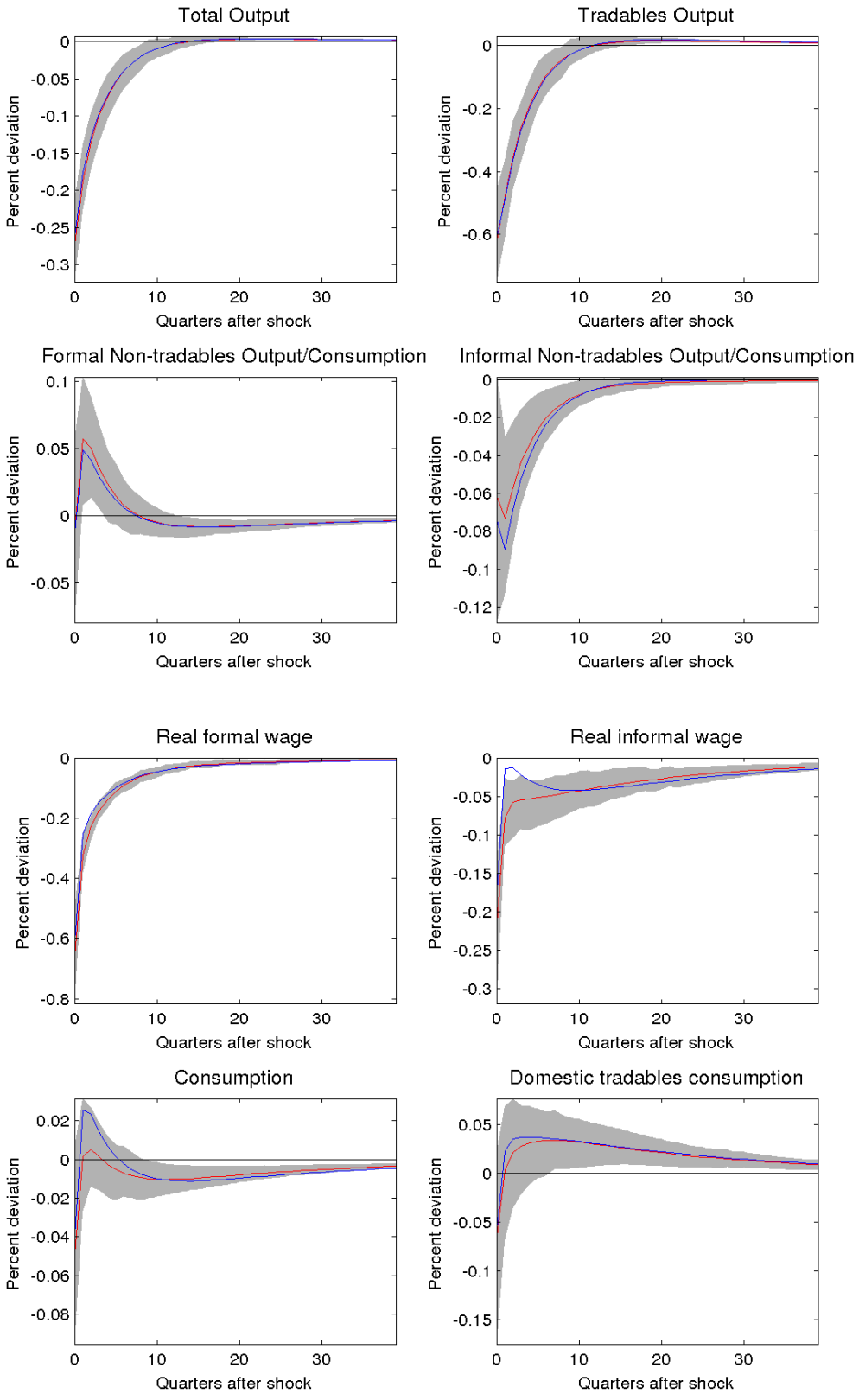


FIGURE C.10: Interest rate shock (2)

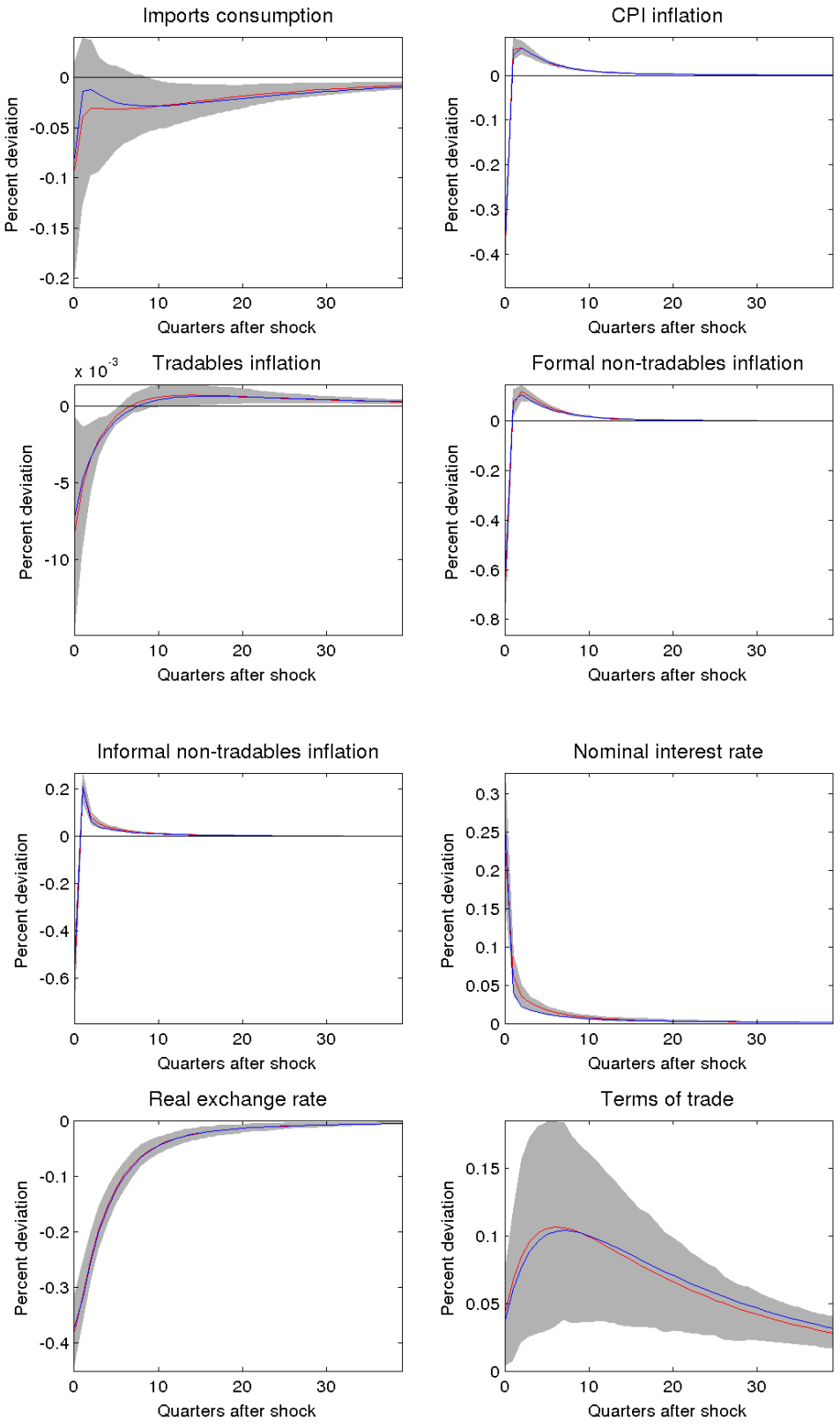


FIGURE C.11: Imported inflation shock (1)

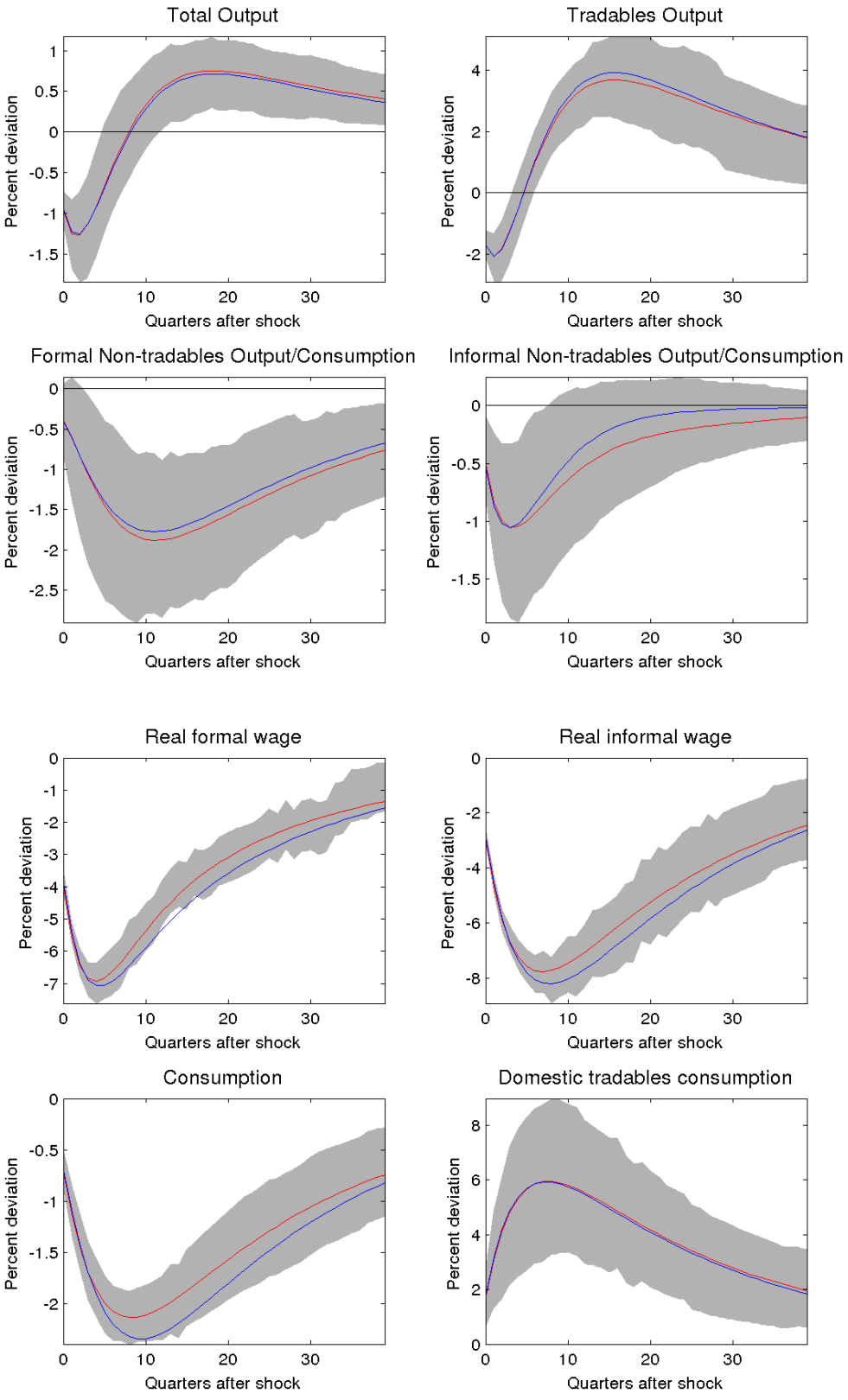


FIGURE C.12: Imported inflation shock (2)

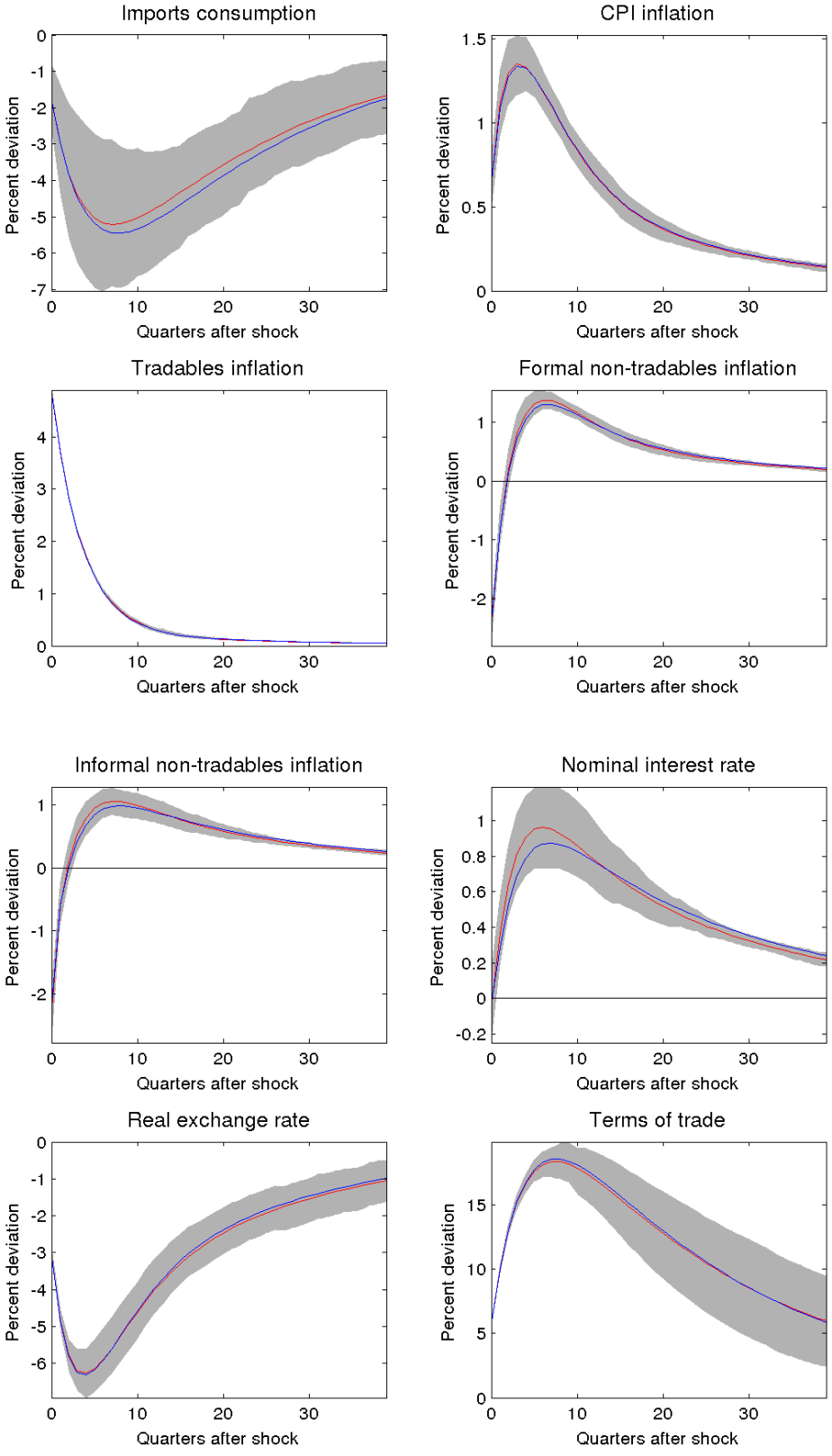


FIGURE C.13: Foreign demand shock (1)

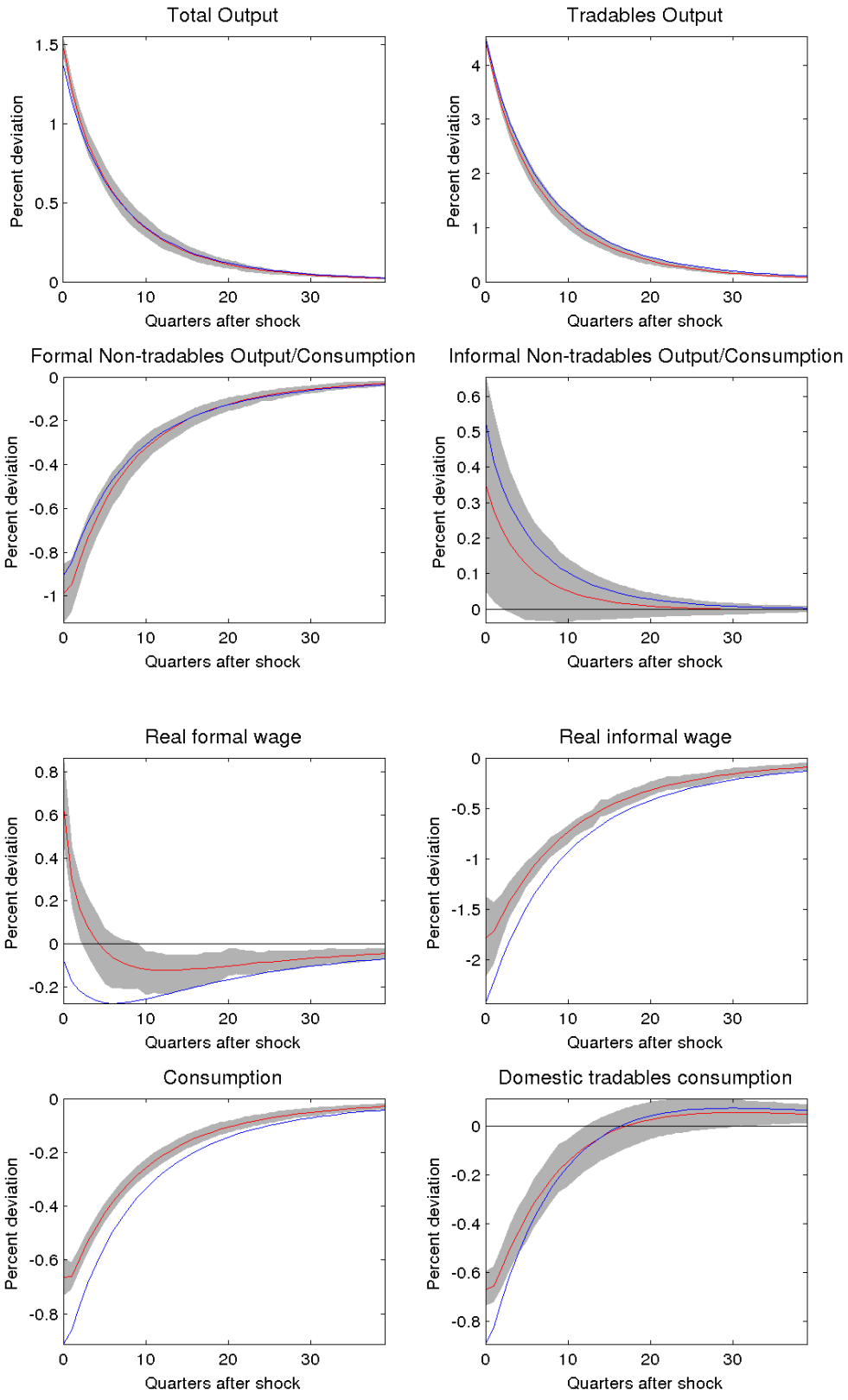
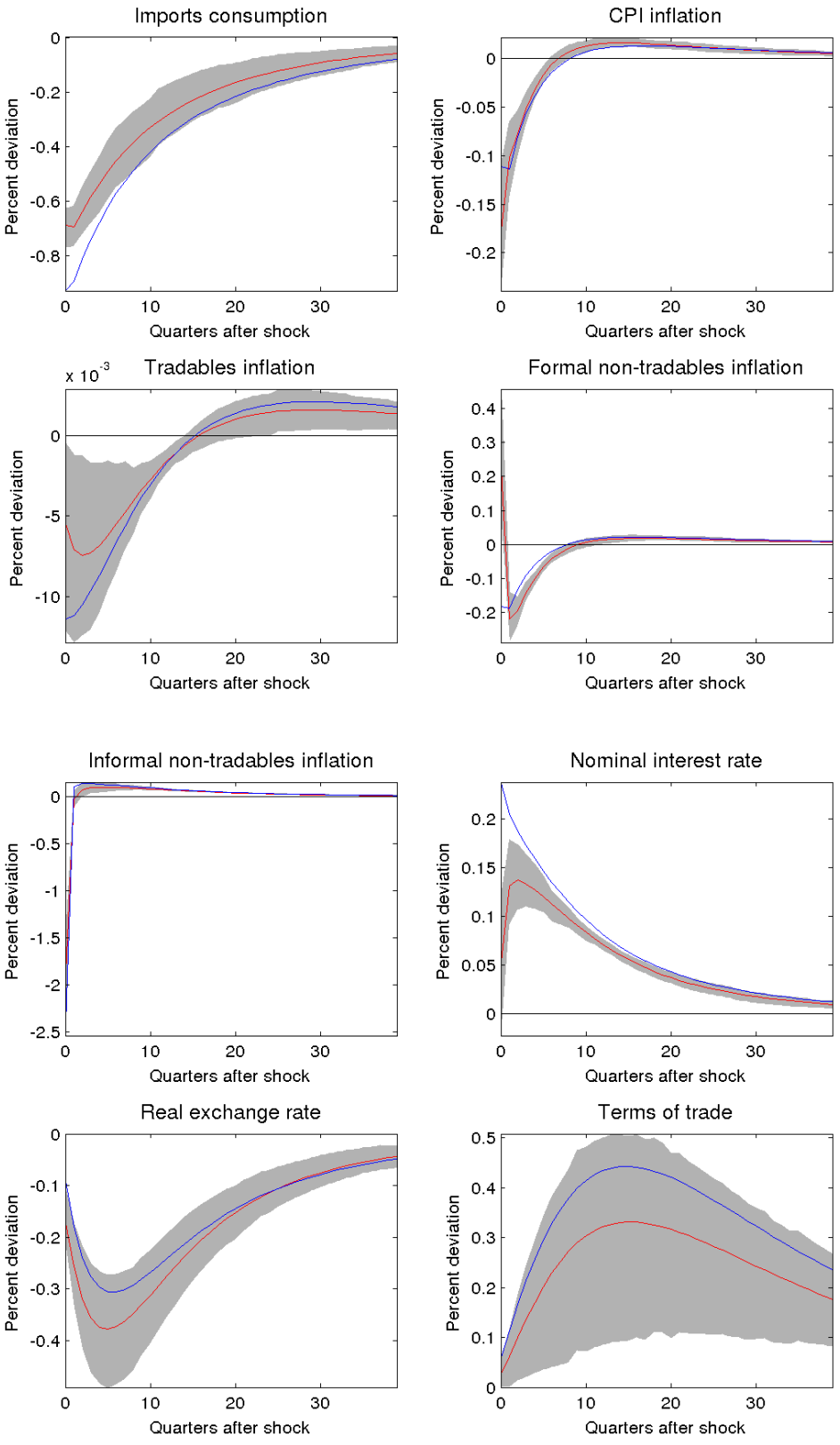


FIGURE C.14: Foreign demand shock (2)



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